QUANTUM COMPLEXITY & ERROR CORRECTION

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joint works with

Anurag Anshu (Harvard)   Chinmay Nirkhe (IBM Research)

Quynh Nguyen (Harvard)

INTRODUCTION

General theme: interplay between

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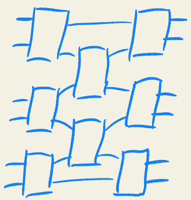
complexity theory



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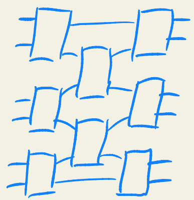

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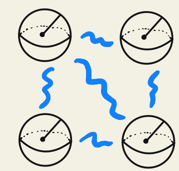

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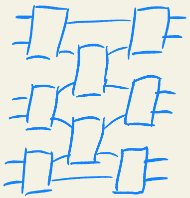

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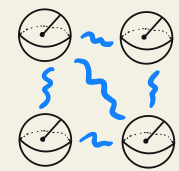

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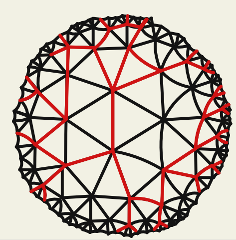

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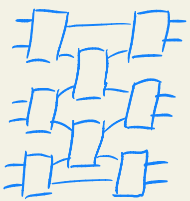

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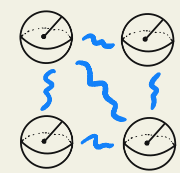

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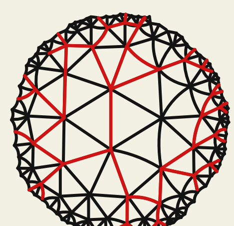

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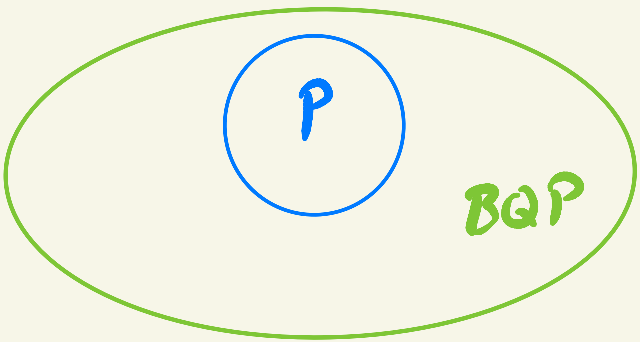

solution can be in polynomial time

computed

classically	P
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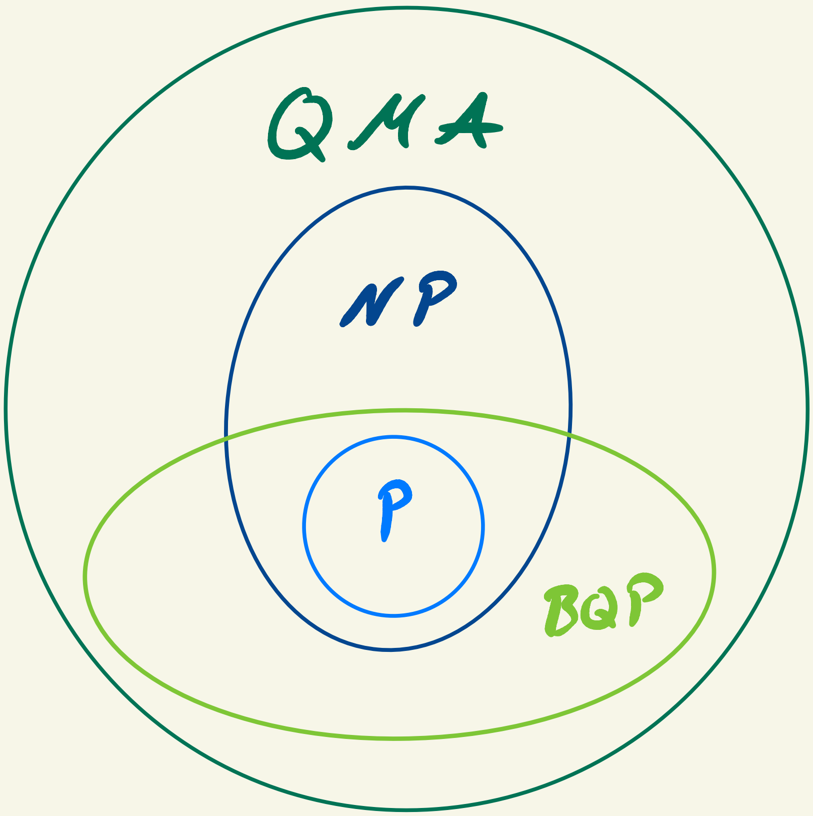

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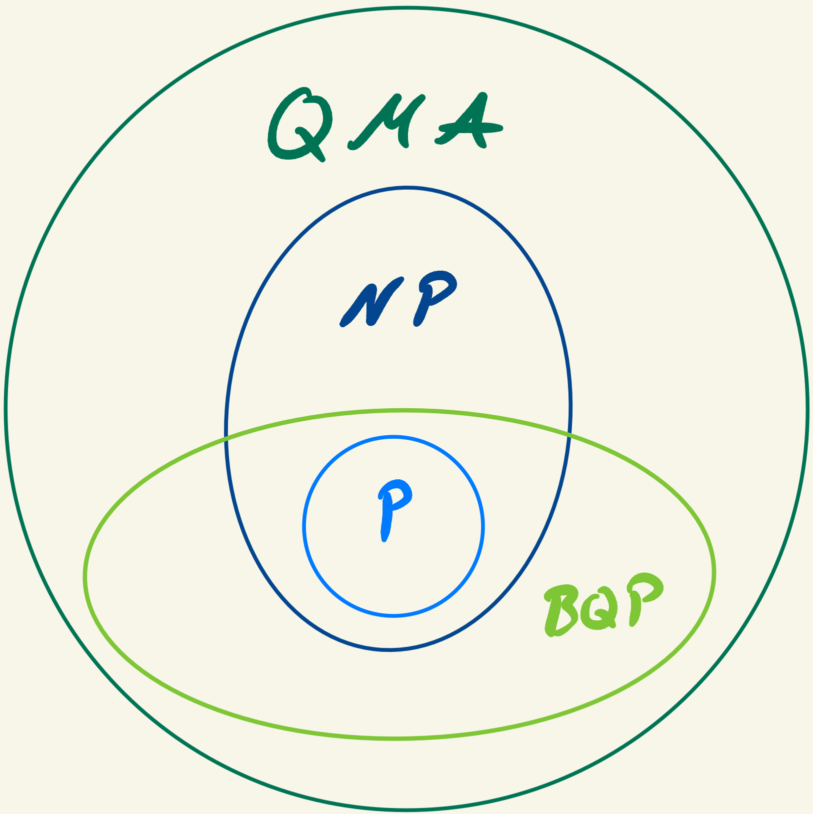

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w.l.o.g. bit-string $x \in \{0,1\}^*$

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EXAMPLES:

- factorization of a number — prime factors
- proof of a mathematical theorem — proof
- satisfyability of boolean formulas — satisfying assignment of variables

NP- COMPLETENESS

problems that belong to NP that are so general that they capture the structure of all problems in NP

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Trivial Example : WITNESS EXISTENCE

given

$$x = \langle \underbrace{M}_{\text{Turing Machine}}, \underbrace{y}_{\in \{0,1\}^*}, \underbrace{p}_{\text{polynomial}} \rangle \in \{0,1\}^*$$

decide whether there exists a string $w \in \{0,1\}^*$, $|w| \leq p(|y|)$ such that M accepts input (y, w) after $p(|y|)$ or fewer steps or not

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problems that belong to NP that are so general that they capture the structure of all problems in NP

Non-trivial example : CONSTRAINED SATISFACTION PROBLEMS (CSPs)

given set of m local (sparse) checks on n bit-variables

$$\{ C_i(x) = x_{i_1} \oplus x_{i_2} \oplus x_{i_3} \} \longrightarrow C(x) = \sum_{i=1}^{m} C_i(x)$$

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decide if ① exists $x \in \{0,1\}^n$: $C(x) = 0$

② for all $x \in \{0,1\}^n$: $C(x) \geq 1$

EXTENDING NOTIONS OF PROOF

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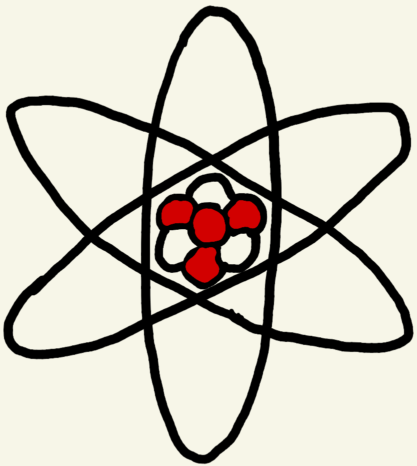

QUANTUM

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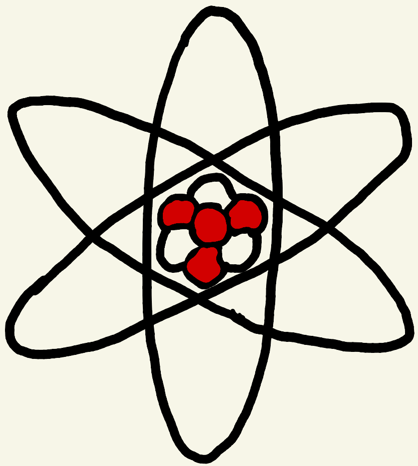

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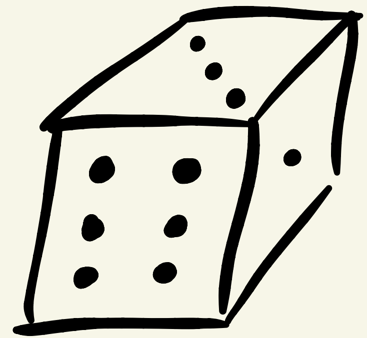
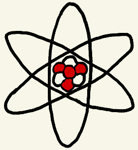


PROBABILISTIC

1 MAKE PROOFS QUANTUM 

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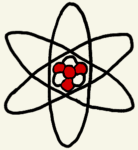

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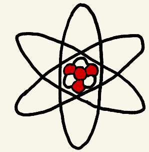
1 MAKE PROOFS QUANTUM 

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poly-time quantum computation*

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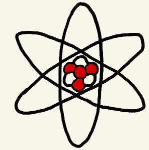

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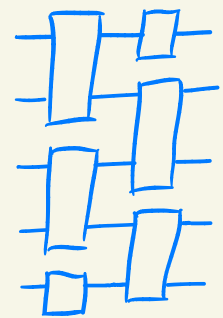

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classical description can be generated in poly(n)-time by TM

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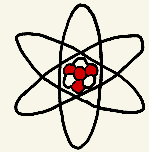

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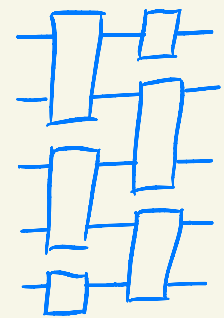

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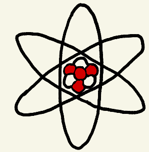

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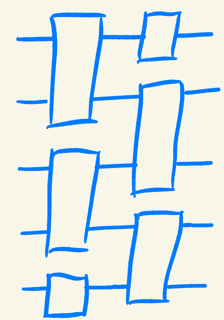

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The class of problems of this form : QMA [KITAEV '99]

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QMA-complete problem: LOCAL HAMILTONIAN

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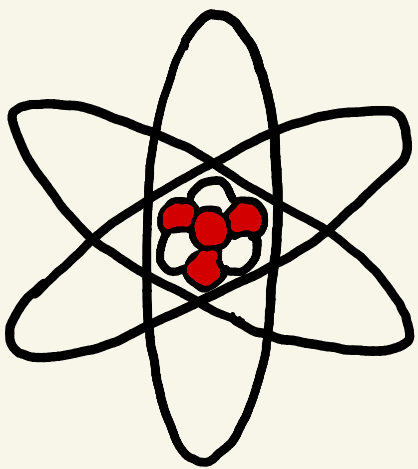

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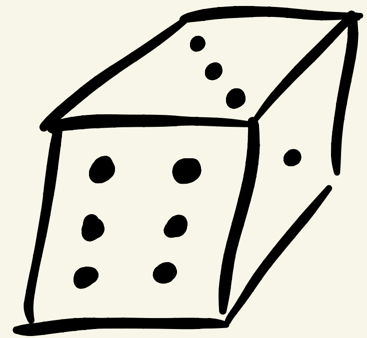

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Probabilistically Checkable Proofs (PCPs)

No need to know full proof $\Rightarrow$ suffices to be given $O(1)$ random bits

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PCP THEOREM [Arora, Safra, Lund, Motwani, Sudan, Szegedy; Dinur]

Every NP problem is equivalent to a problem for which we only require access to $O(1)$ bits

accept valid proofs with probability 1

accept invalid proofs with probability 1/2

② MAKE PROOFS PROBABILISTIC

PCP theorem implies for CSPs:

NP-hard to decide if ① $\exists\, x \in \{0,1\}^n : C(x) = 0$

② $\forall\, x \in \{0,1\}^n : C(x) \geq \frac{m}{2}$

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$\Rightarrow$ WE CAN HAVE "APPROXIMATE" PROOFS

Any x s.t. $C(x) < \frac{m}{4}$ can be probabilistically verified

THE qPCP CONJECTURE

NP

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NP → QMA

NP → PCP

THE qPCP CONJECTURE

$$\begin{tikzcd} & \mathrm{QMA} \arrow[dr] & \\ \mathrm{NP} \arrow[ur] \arrow[dr] & & \mathrm{qPCP} \\ & \mathrm{PCP} \arrow[ur] & \end{tikzcd}$$

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For LOCAL HAMILTONIAN this implies that it is QMA-hard to decide whether

① $\exists\, |\psi\rangle$ s.t. $\langle\psi|H|\psi\rangle = 0$

② $\forall\, |\psi\rangle$: $\langle\psi|H|\psi\rangle \geq \varepsilon m$

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$\Rightarrow$ every state of energy $\leq \frac{\varepsilon m}{2}$ is a valid proof

NO LOW-ENERGY TRIVIAL STATES

$\boxed{\text{Approximate proofs } \psi \text{ should be complex}}$

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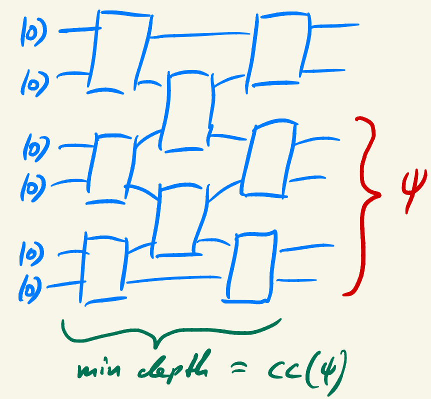
Approximate proofs ψ should be complex

Measured by circuit complexity $cc(\psi)$:

minimum depth of quantum circuit consisting of 2-qubit gates that prepares ψ from $|0\ldots0\rangle$



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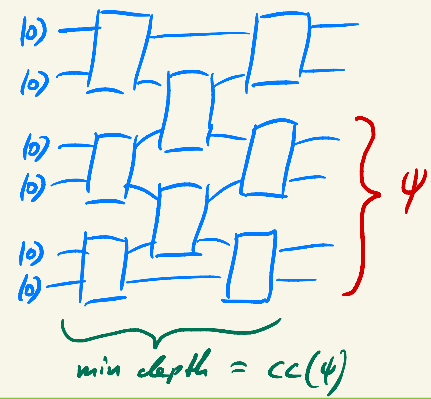

NLTS Conjectured [FREEDMAN, HASTINGS '14]

$\exists$ universal constant $\varepsilon > 0$ & family of Hamiltonians $\{H^{(n)}\}_n$ on n qubits s.t.

$\forall$ $\psi^{(n)}$ with $\mathrm{tr}(H^{(n)}\psi^{(n)}) < \varepsilon n$ it holds that

$$cc(\psi^{(n)}) \geq \Omega(\log n) \geq \omega(1) \quad \text{(super-constant)}$$

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define $E = tr(H\psi)$

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by $\neg NLTS$ $\exists$ trivial state ψ such that

if $E=0 \Rightarrow$ can classically check $tr(H\psi) = \sum_{i=1}^{m} tr(h_i \psi) \leq \frac{\varepsilon m}{2}$

else $E \geq \varepsilon m \Rightarrow \forall\, U$: $tr(H\psi) \geq \varepsilon m$

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Have a classical proof + verification for QMA problem $\lightning$ assuming $NP \neq QMA$

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can classically check $\mathrm{Tr}(H\psi) \leq \frac{\varepsilon m}{2}$

LIGHT-CONE ARGUMENT

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For any operator A

$\mathrm{tr}\left(A \ \mathrm{tr}_s\left(U^\dagger \mathfrak{z} U\right)\right)$

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can classically check $\mathrm{tr}(H\rho) \le \frac{\varepsilon m}{2}$

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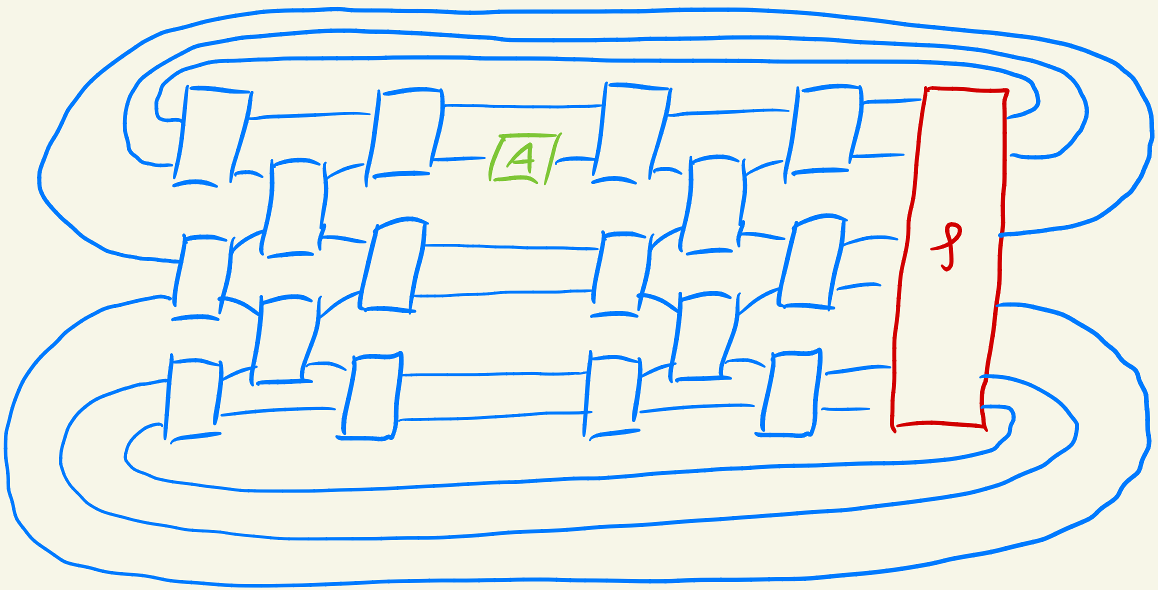

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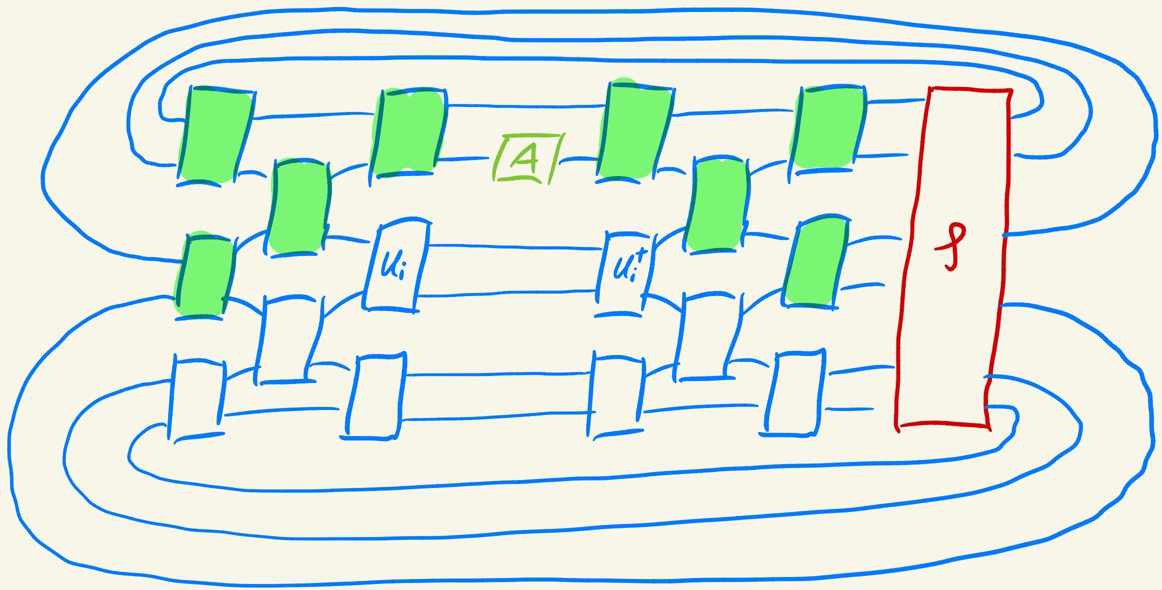

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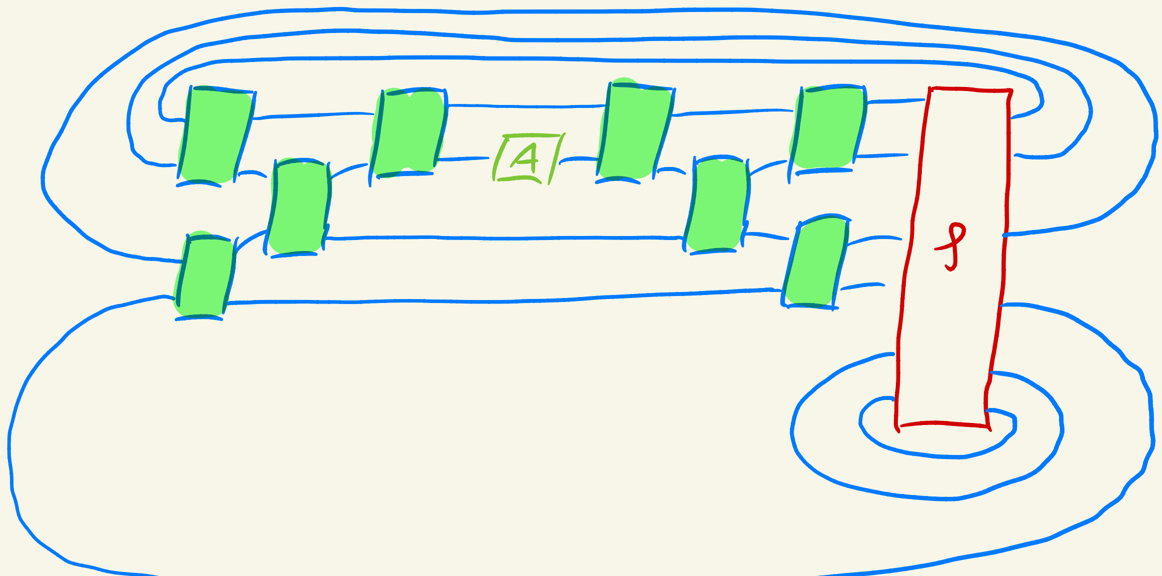

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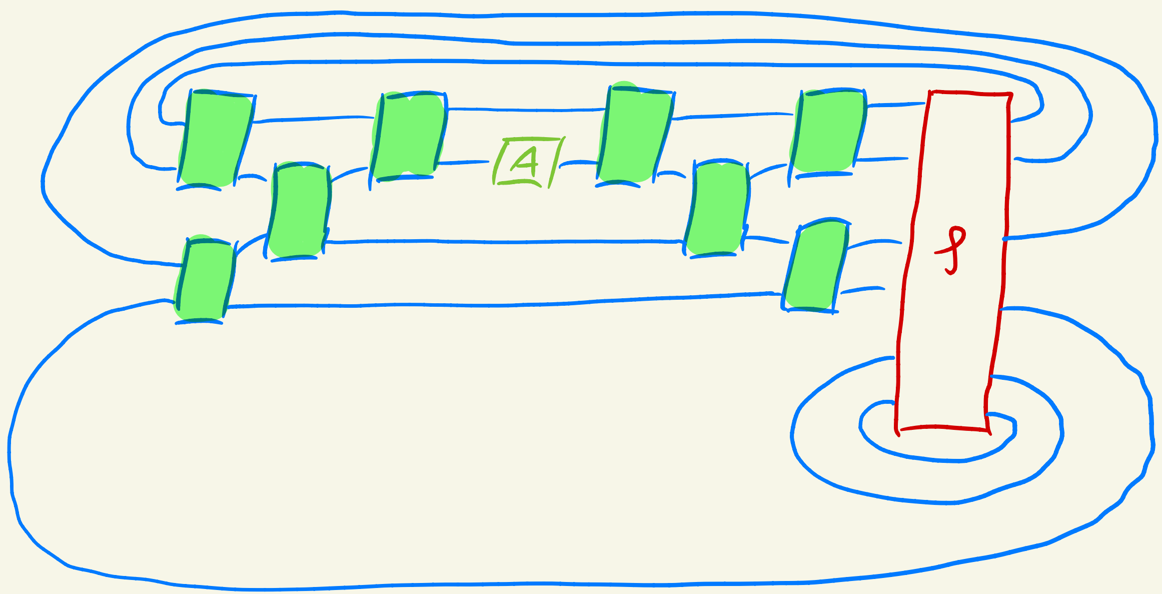
$\mathrm{tr}(U A U^\dagger \rho_L)$ ⟸



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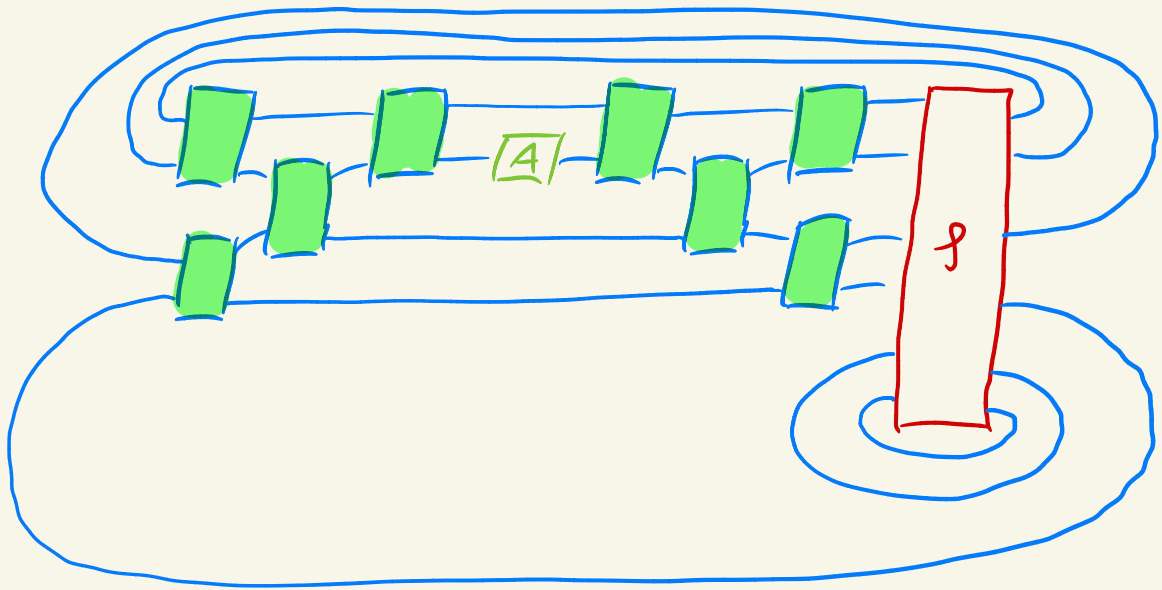

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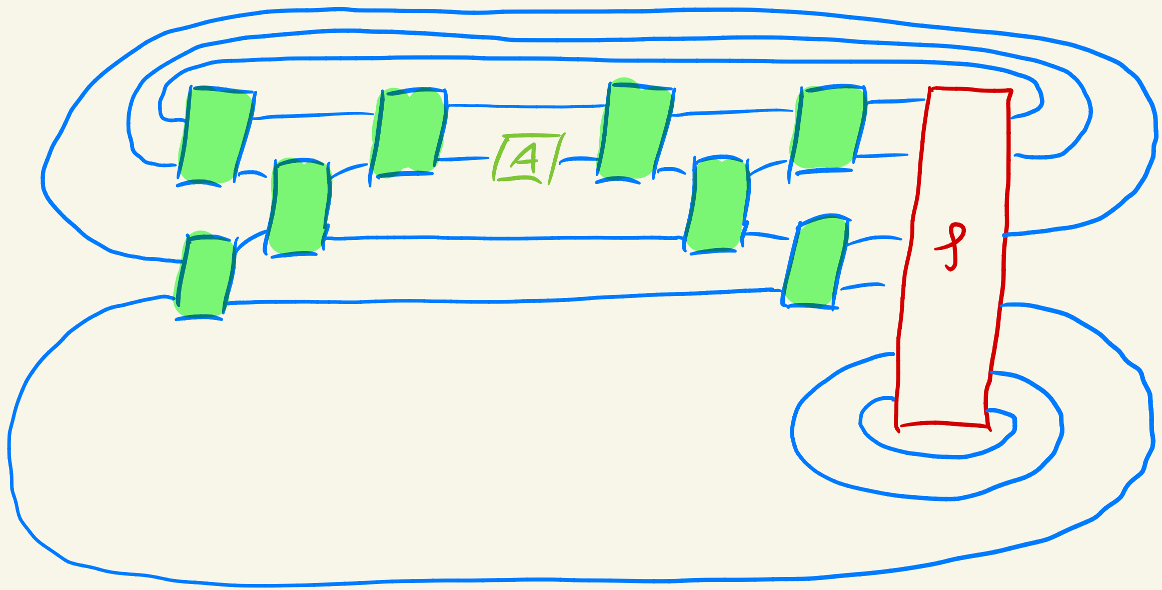

$\mathrm{tr}(U A U^\dagger \rho_L)$ ⟸

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can be computed efficiently when $|L| \in O(1)$

NO LOW-ENERGY TRIVIAL STATES

NLTS conjecture [FREEDMAN, HASTINGS '14]

There exist families of local Hamiltonians s.t.

no low-energy state is the output of a $O(1)$-depth circuit.

If the conjecture had been false,

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NLTS theorem [ANSHU, NPB, NIRKHE '22]
Good qLDPC codes give rise to NLTS Hamiltonians.

IMPLICATION FOR MANY-BODY PHYSICS

Fact of Life: Quantum mechanics does not apply to our scale

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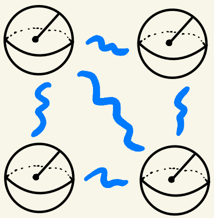

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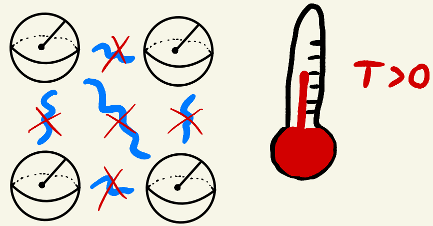

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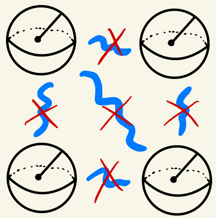

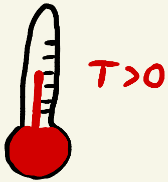

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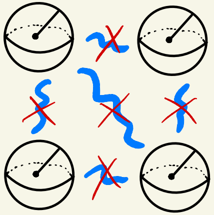

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NLTS $\Rightarrow$ $\exists$ physical system where Gibbs state is non-trivial

$$cc\left(e^{-\beta H}/Z\right) \geq \Omega(\log n)$$

Therefore: non-vanishing, global entanglement!

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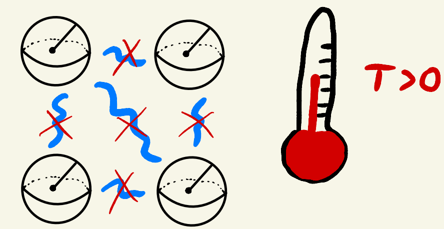

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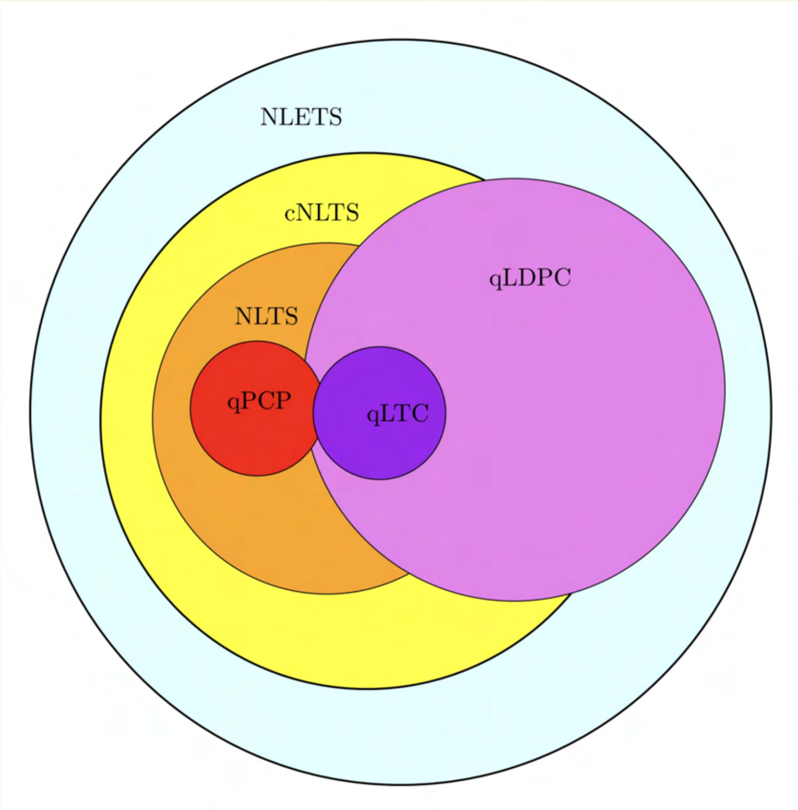
Complexity theory useful for many-body physics!

ROBUST ENTANGLEMENT ZOO



[ELDAR & HARROW '17]

Several interrelated conjectures regarding the robustness of entanglement

1. NLTS - There exist local Hamiltonians such that any low-energy state is non-trivial.
2. cNLTS - There exist local Hamiltonians such that any quantum state satisfying a $\geq 1-\varepsilon$ fraction of all local terms is non-trivial.
3. NLETS - There exist local Hamiltonians such that any quantum state that is equal to a ground state up to a unitary incident on at most an ε fraction of qubits, is non-trivial.
4. qLTC - There exist local Hamiltonians for which the energy of a quantum state is proportional to its distance from the ground-space of the Hamiltonian.
5. qLDPC - There exist quantum codes with local checks, and minimal distance scaling linearly in the number of qubits.
6. qPCP - It is as hard to approximate the ground energy of a local Hamiltonian to a constant fraction accuracy, as it is to estimate it to inverse-polynomial accuracy.

NLETS proved [ELDAR, HARROW '17] [NIRKHE, VAZIRANI, YUEN '18]

[ANSHU, NIRKHE '22]

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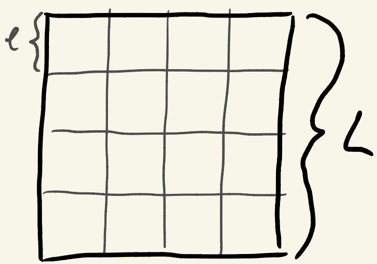
can prepare constant size patches $\ell \times \ell$ using constant depth circuit

$\Rightarrow$ $|\psi\rangle$ is a trivial state



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$E \sim \ell \cdot \frac{L^2}{\ell^2} \sim \frac{1}{\ell} n$

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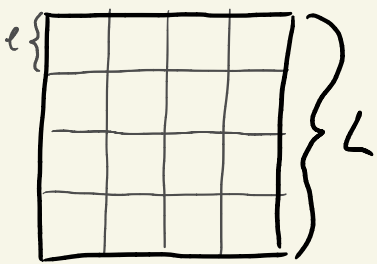

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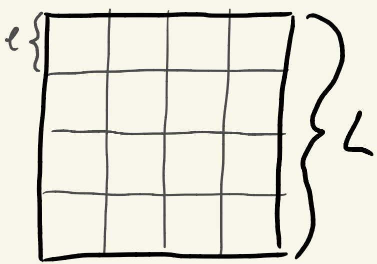

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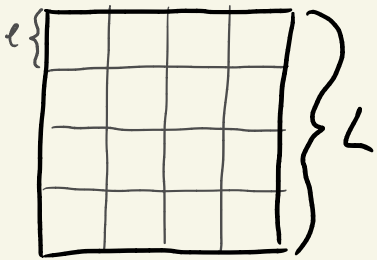

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[AHARONOV, ARAD, VIDICK '13]

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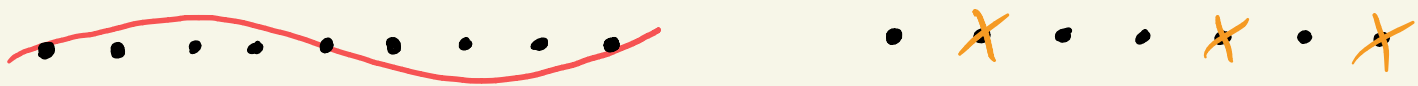

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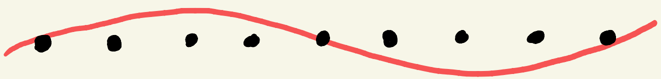

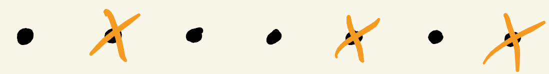

$\boxed{\text{cNLTS Hamiltonians exist [ANSHU, NPB '22]}}$

Journal of Mathematical Physics 63 (12) 2022

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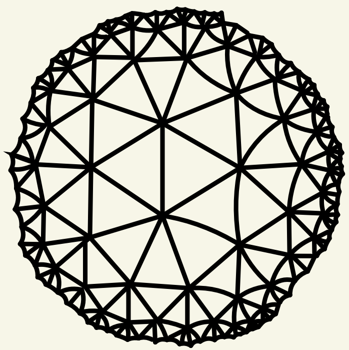

Expander graphs : well-connected & sparse families of graphs

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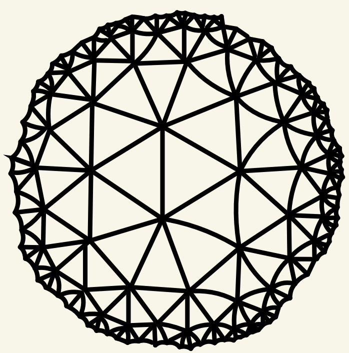

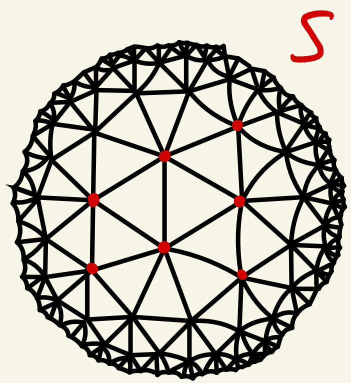

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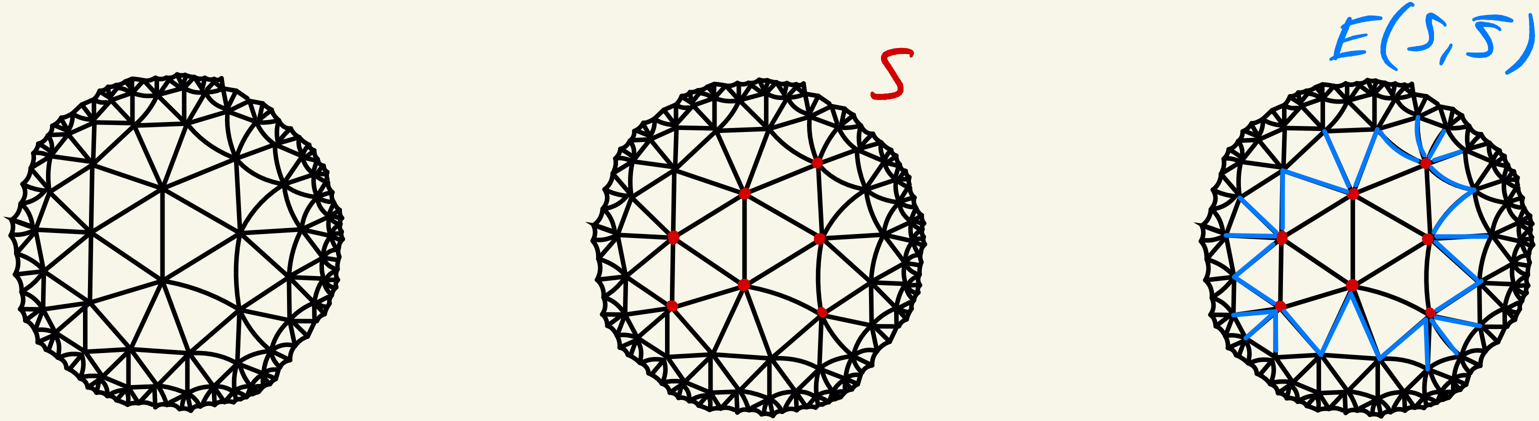
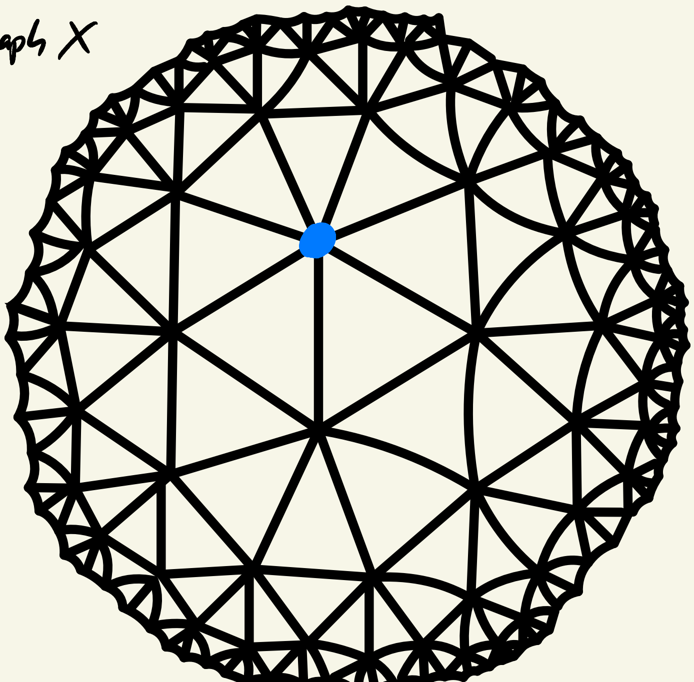


Expander graphs: $\underbrace{\text{well-connected}}_{\min\limits_{0<|S|<\frac{|V|}{2}} \frac{|E(S,\bar{S})|}{|S|}}$ & sparse families of graphs

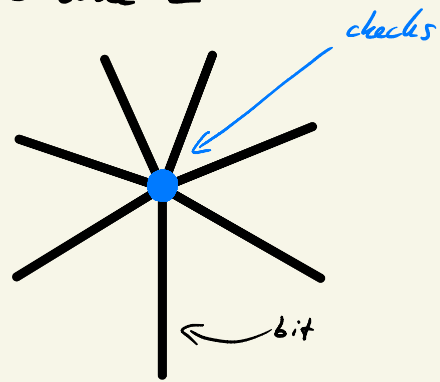
Combinatorial NLTS (cNLTS)

SIPSER, SPIELMAN '95 — Expander graphs give good LDPC codes

graph X



"local code" L



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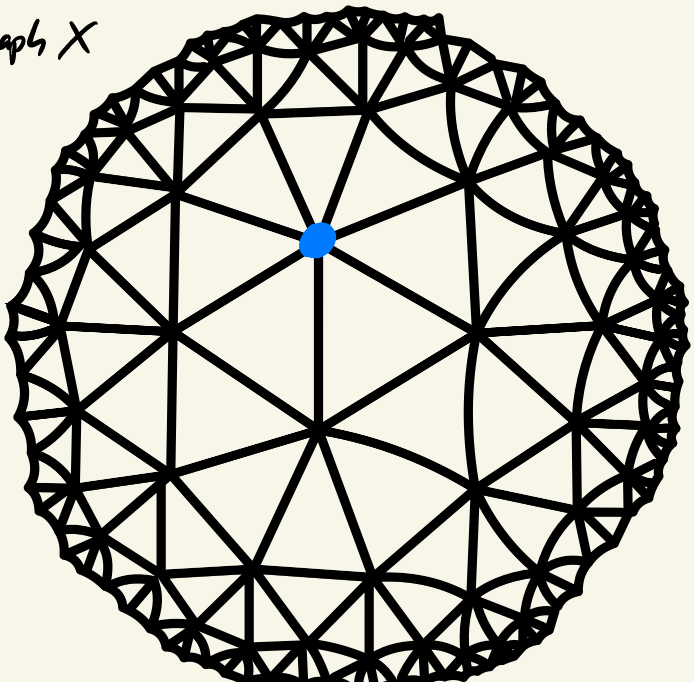

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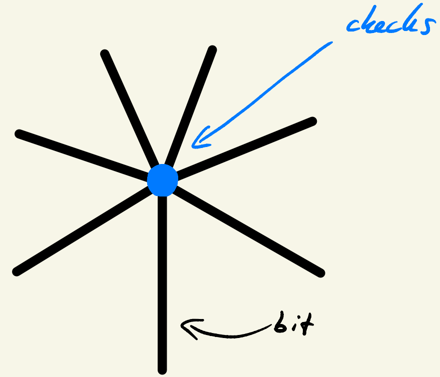
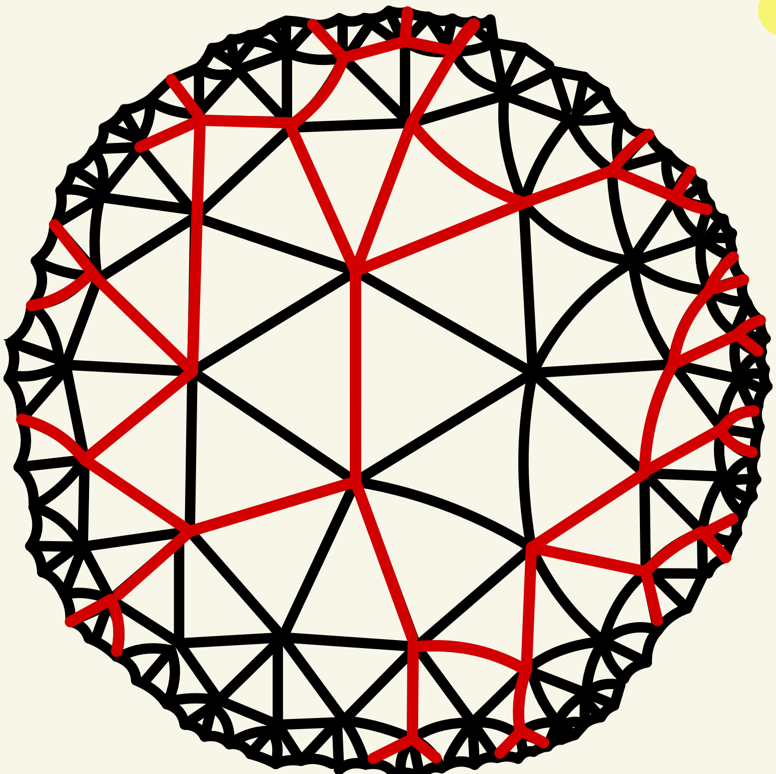


MESHULAM arXiv:1803.05643

"Homology with local coefficients"

$C(X, L)$

Combinatorial NLTS (cNLTS)



$C(X, L)$ has linear distance

Local code has distance $d = 3$
$\Rightarrow$ either 0 or ≥ 3 incoming edges per vertex

Expansion ensures: distance of global code is $\Theta(n)$

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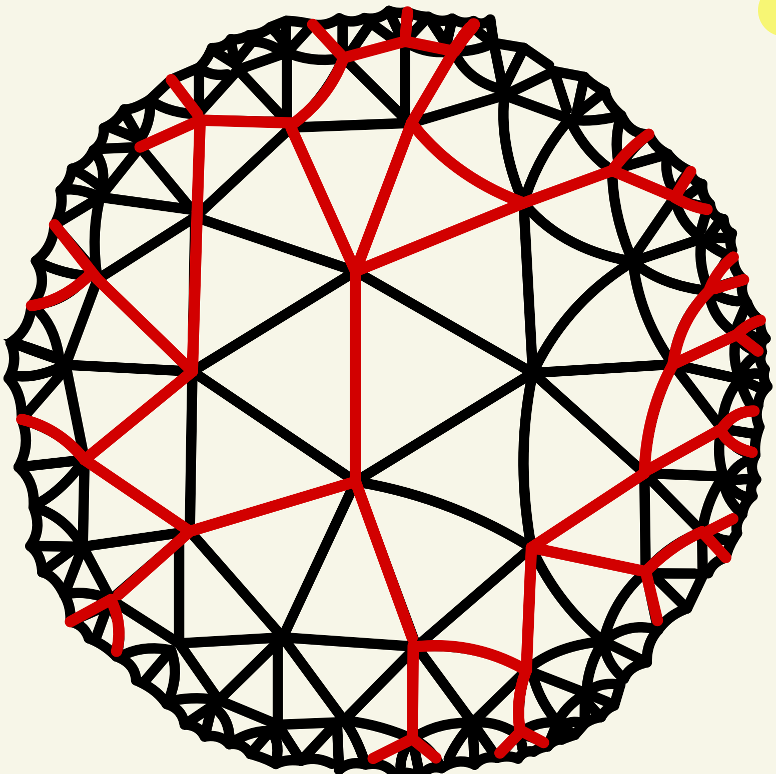

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---

implement expander code as a perturbed tensor network

$\Downarrow$

local Hamiltonian

ground state determined by $C(X,L)$

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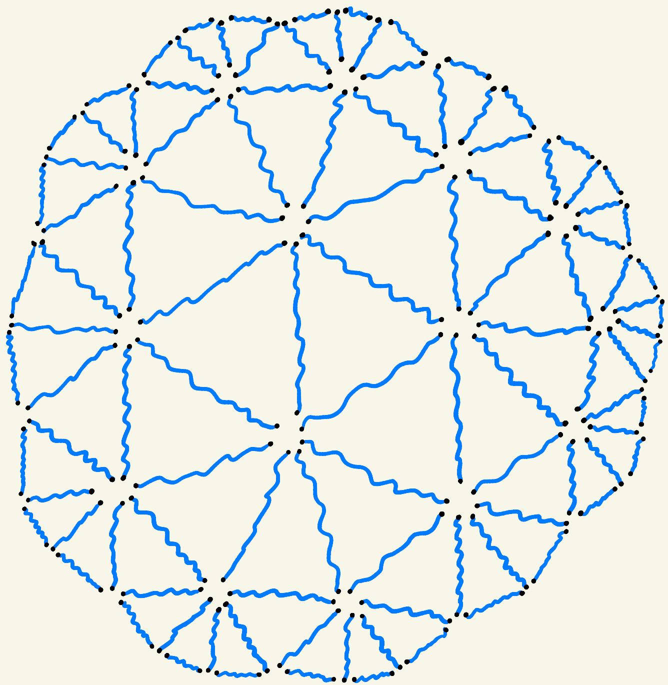

Perturbed tensor network is a projected entangled pair state (PEPS)

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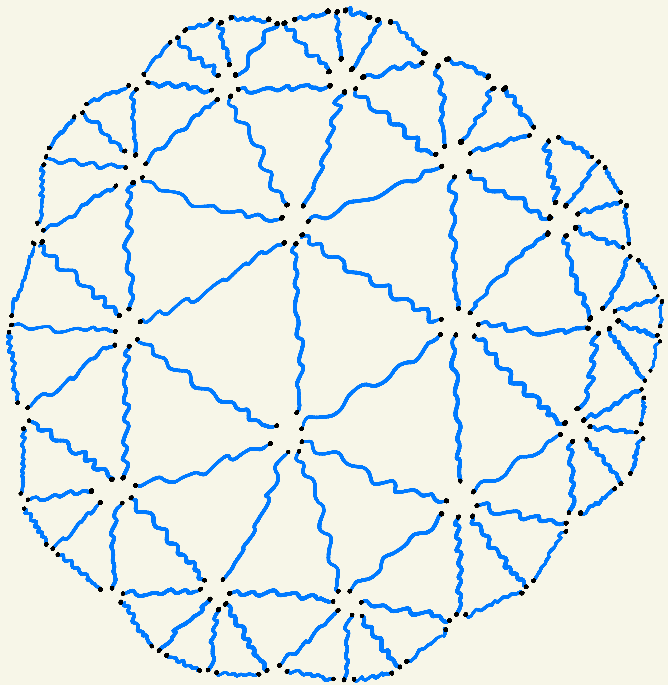

Perturbed tensor network is a projected entangled pair state (PEPS)

① replace every edge / bit with an EPR pair

$$\bullet\!\!\sim\!\!\sim\!\!\sim\!\!\bullet \; = \frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right)$$

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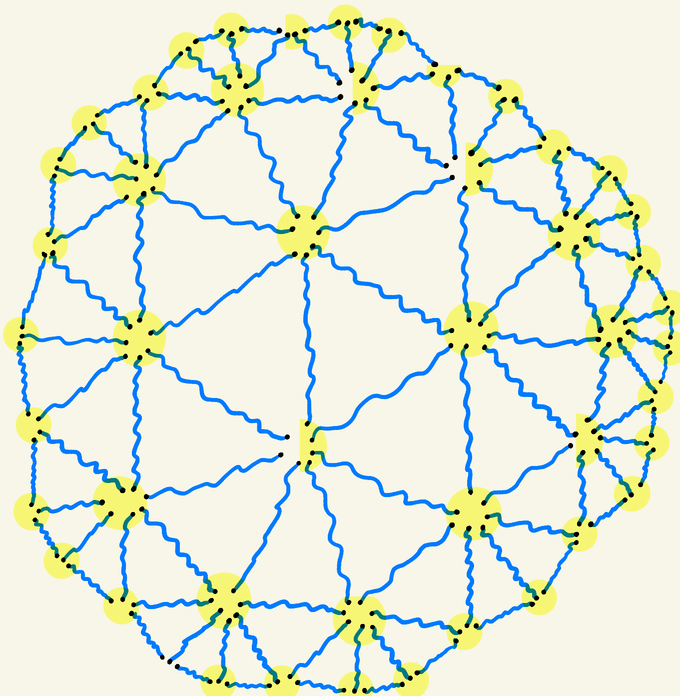

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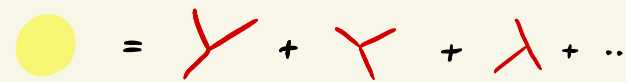

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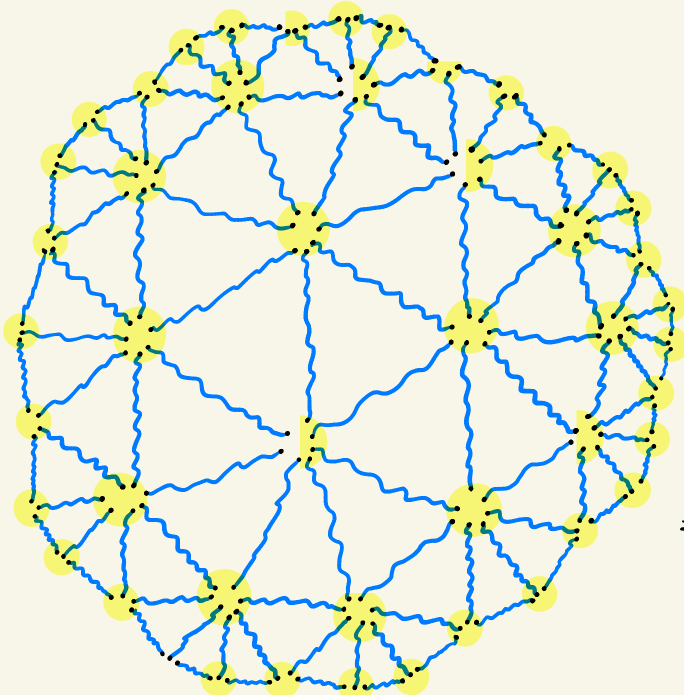

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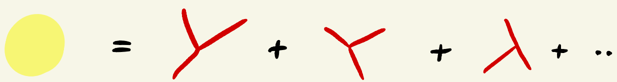
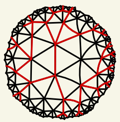
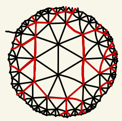


$\Rightarrow$ state is uniform superposition over code words

$\frac{1}{\sqrt{2^k}}$  $+ \frac{1}{\sqrt{2^k}}$  $+ \ldots$

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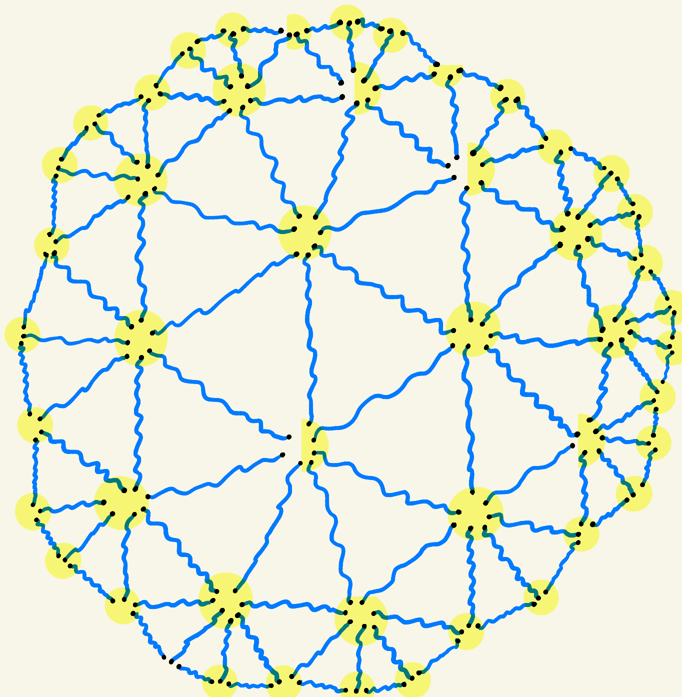
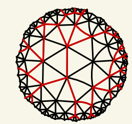
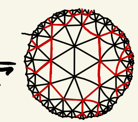


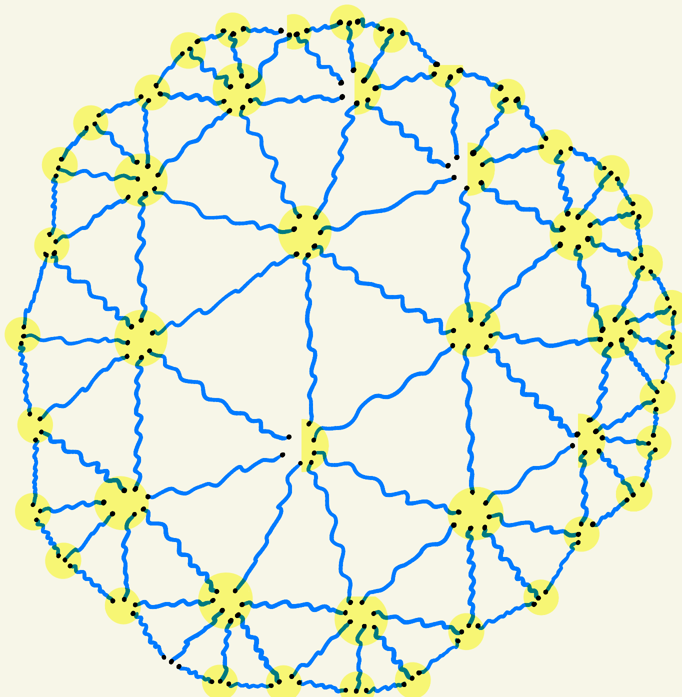
For our construction we require that

$\frac{1}{\sqrt{2^k}}$  $+ \frac{1}{\sqrt{2^k}}$  $+ \dots$

is the UNIQUE ground state

of a Hamiltonian

Combinatorial NLTS (cNLTS)



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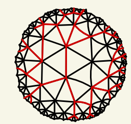
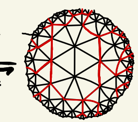
$\frac{1}{\sqrt{2^k}}$  $+ \frac{1}{\sqrt{2^k}}$  $+ \ldots$

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$\Rightarrow$ solve this by perturbing each projector $+ \delta I$

$\Rightarrow$ $H = \sum_i h_i$

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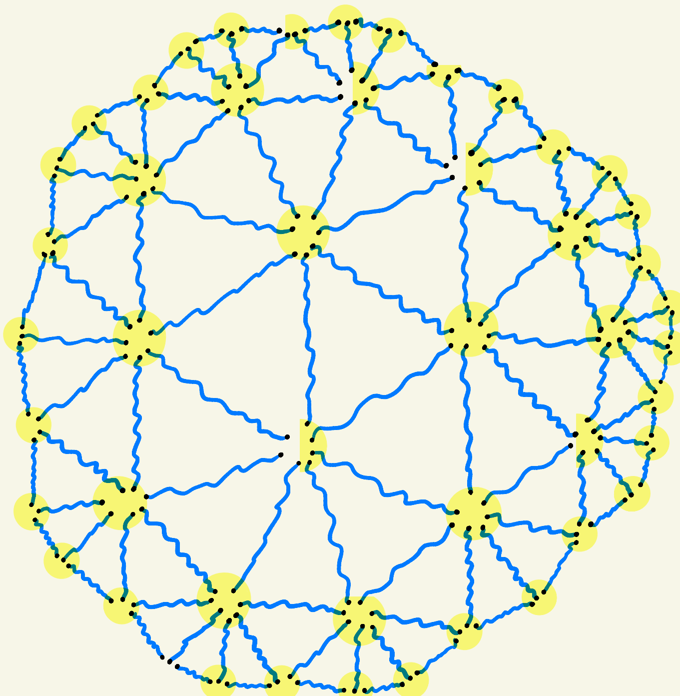

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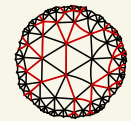
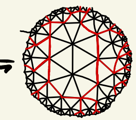
$\frac{1}{\sqrt{2^k}}$  $+ \frac{1}{\sqrt{2^k}}$  $+ \dots$

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$\Rightarrow$ solve this by perturbing each projector $+ \delta I$

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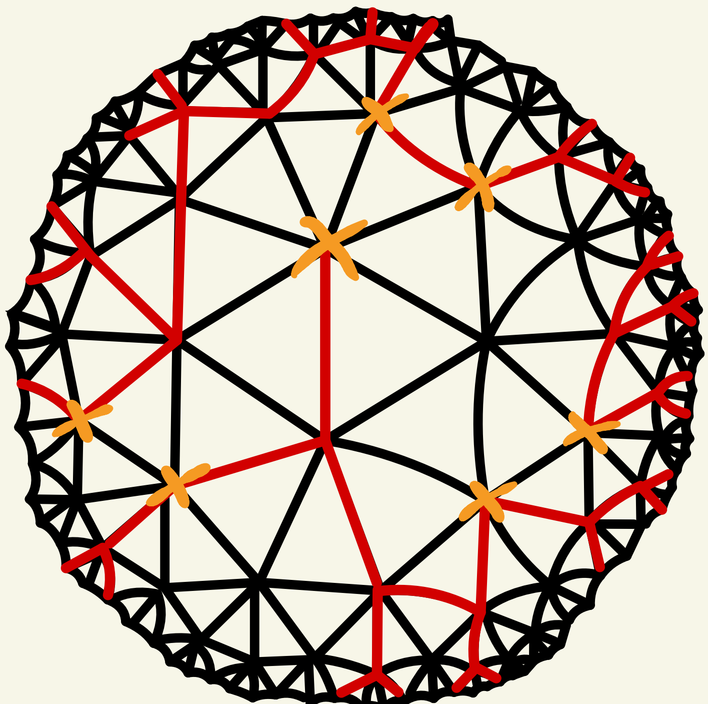
THIS WILL BE IMPORTANT LATER

Combinatorial NLTS (cNLTS)

states violating few terms $\quad |\{ i \mid \mathrm{tr}(h_i \psi) > 0 \}| < \varepsilon m$



vertices where Gauß law is violated ✗

(no code word of L around those vertices)

ε-approximate code words of global code $C(X, L)$

Combinatorial NLTS (cNLTS)

Observation: approximate code words of codes $C(X, L)$

are <u>either</u> close <u>or</u> far apart (in terms of Hamming distance)

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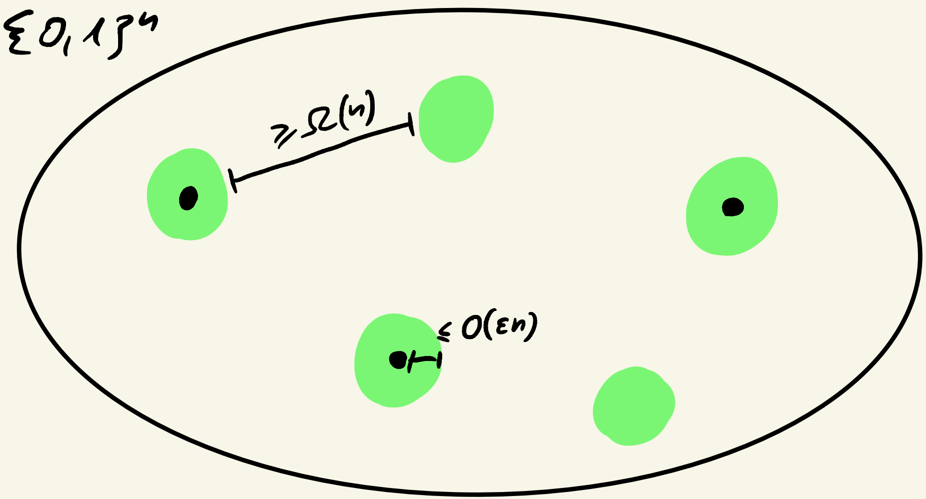
$\{0,1\}^n$



$\bullet \,\hat{=}$ codewords

(green disc) $\hat{=}$ states that violate $\leq \varepsilon m$ checks

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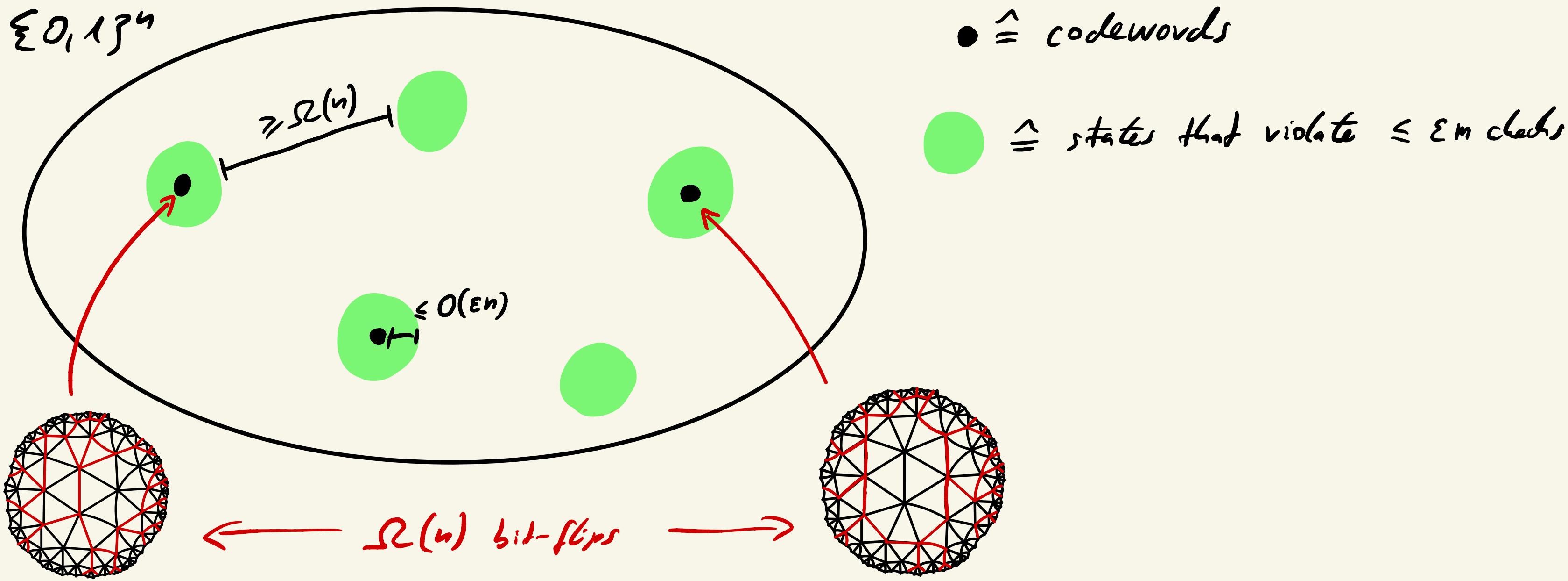
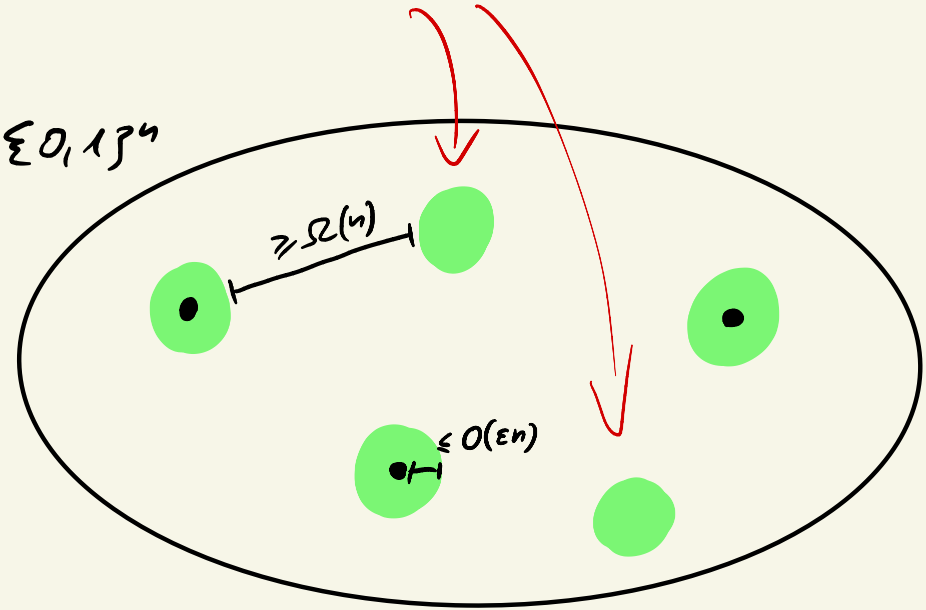


Combinatorial NLTS (cNLTS)

Clusters of approximate code words
without a code word in them



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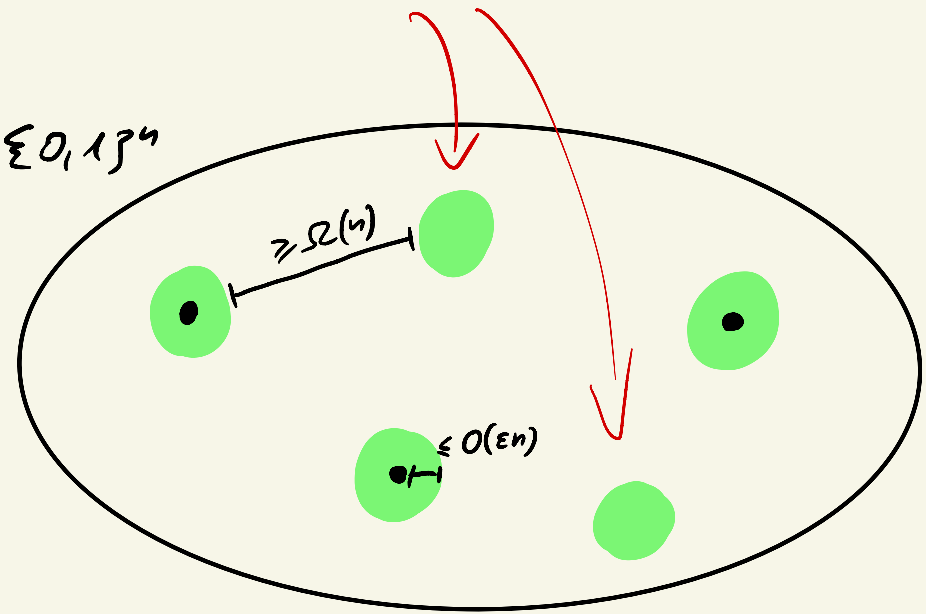

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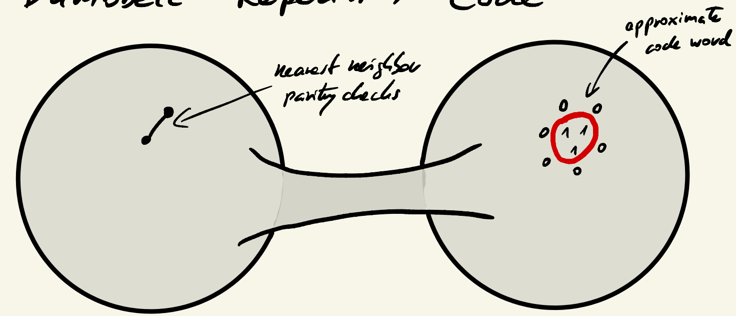
EXAMPLE:

"Dumbbell Repetition Code"



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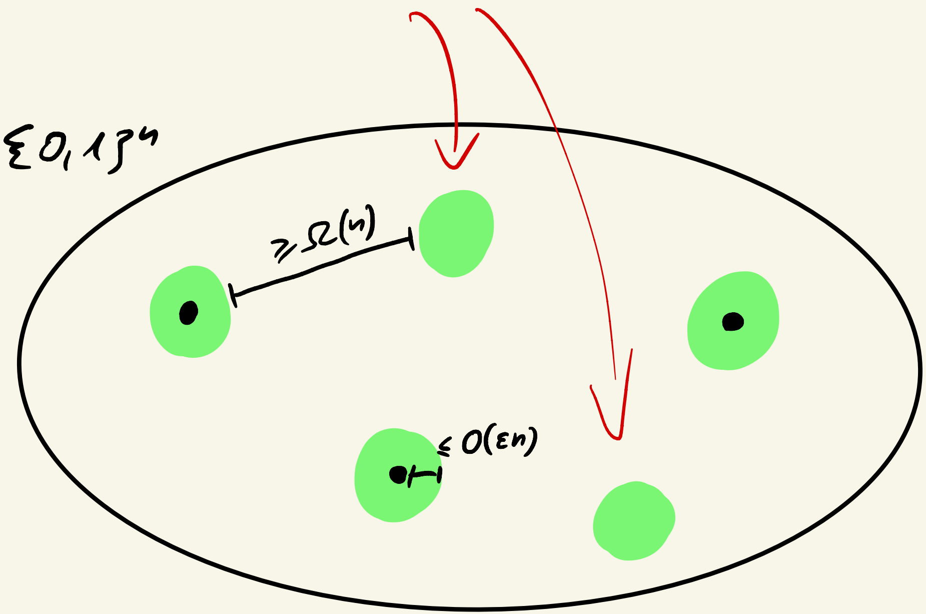

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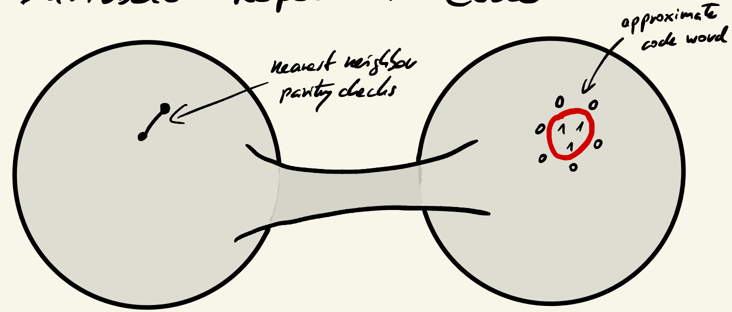
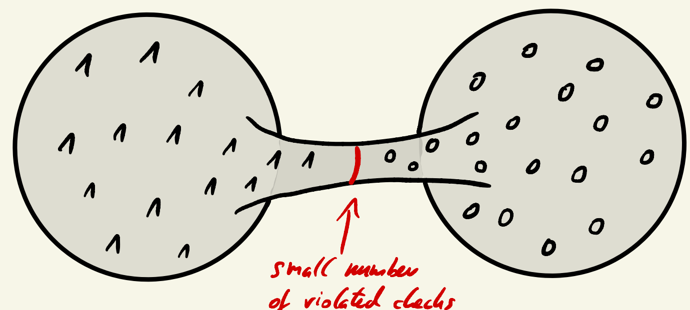


Setting one side 1 & other side 0 gives approximate code word far from actual code words



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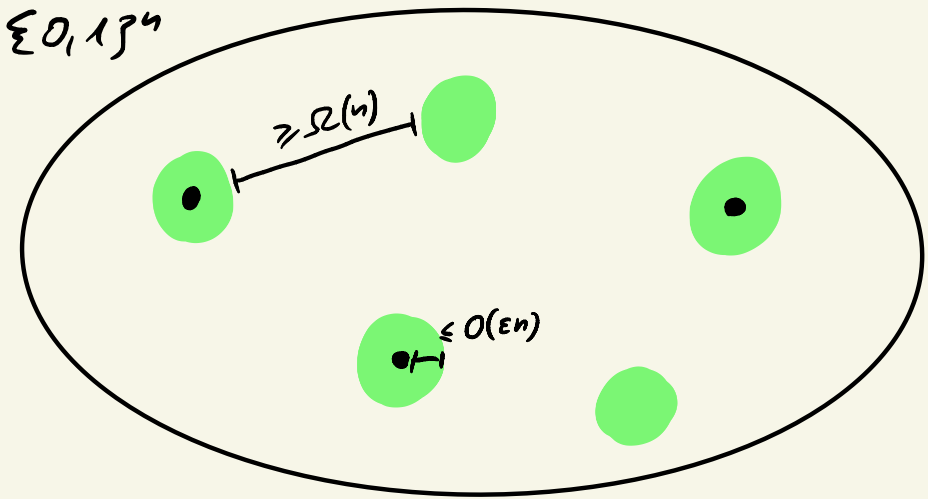


● $\hat{=}$ codewords

● $\hat{=}$ states that violate $\leq \varepsilon m$ checks

PROOF IDEA (same as for NLTS)

- clustering of code words
  $\Rightarrow$ spread of induced prob. dist.
- spread leads to circuit lower bound

~~Combinatorial~~ NLTS

Proof based on the same observation

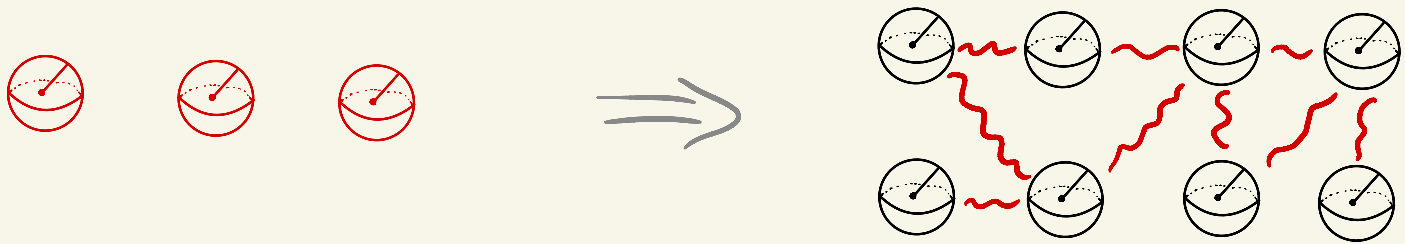
but using a different construction

~~Combinatorial~~ NLTS

Proof based on the same observation
but using a different construction

Quantum Error Correcting Codes

encode k logical qubits into non-local degrees of freedom of
n physical qubits



~~Combinatorial~~ NLTS

Proof based on the same observation
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Quantum Error Correcting Codes

encode k logical qubits into non-local degrees of freedom of
n physical qubits

distance d : minimum number of single-qubit errors that change encoded quantum information

QUANTUM CODES

Shor '94: There are two types of errors we have to correct

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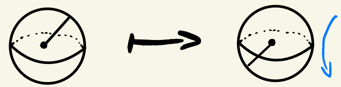
bit-flip / X- errors



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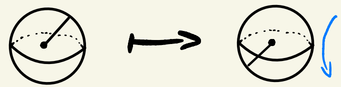

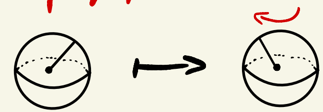

⇝ QUANTUM CODE DEFINED BY TWO PARITY CHECK MATRICES

H_X & H_Z (Heisenberg Uncertainty: $H_X \cdot H_Z^T = 0$)

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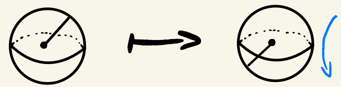

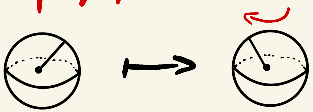

$\rightsquigarrow$ QUANTUM CODE DEFINED BY TWO PARITY CHECK MATRICES

H_X & H_Z (Heisenberg Uncertainty: $H_X \cdot H_Z^T = 0$)

(SPARSE) CHECKS GIVE RISE TO (LOCAL) HAMILTONIAN

GOOD qLDPC CODES

sparse checks

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number of encoded qubits
$k = \Theta(n)$

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code distance

$d = \Theta(n)$

GOOD qLDPC CODES

sparse checks

number of encoded qubits
$k = \Theta(n)$

code distance
$d = \Theta(n)$

Constructed via balanced product of classical codes

Tool loaned from algebraic topology / homology

$\Rightarrow$ somewhat technical

GOOD qLDPC CODES

Instead of algebraic language: use topological intuition

How can we do this?

GOOD qLDPC CODES

Instead of algebraic language: use topological intuition

How can we do this?

Consider repetition code on 3 bits $C = \{000, 111\} = \ker H$

$$H = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix}$$

GOOD qLDPC CODES

Instead of algebraic language : use topological intuition

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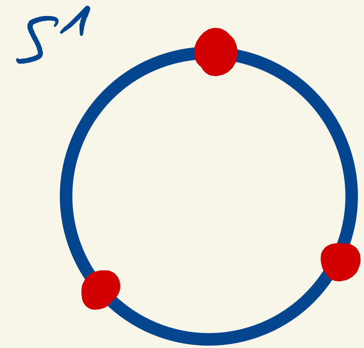
vertex-edge incidence matrix

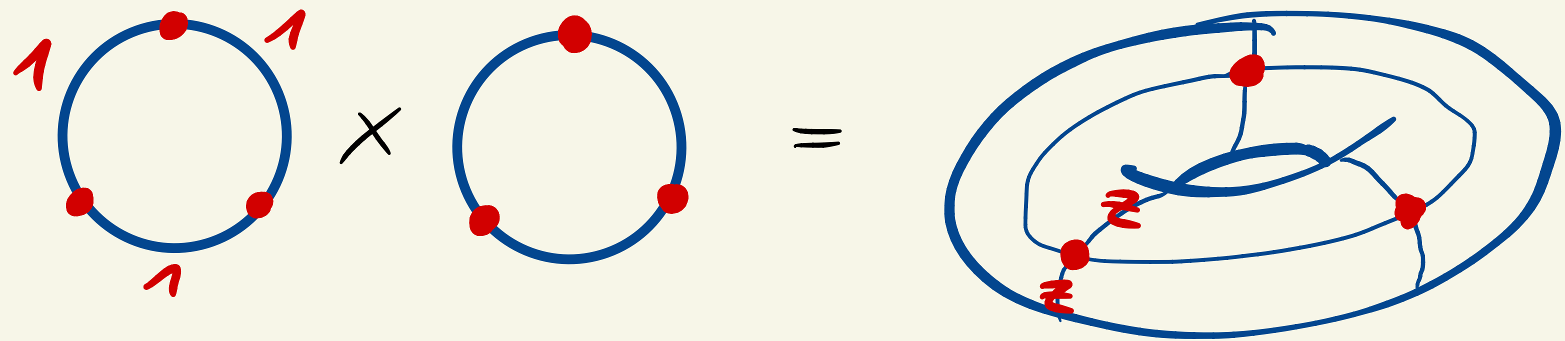


bits $\hat{=}$ edges

checks $\hat{=}$ vertices

Identify rep. code with circle

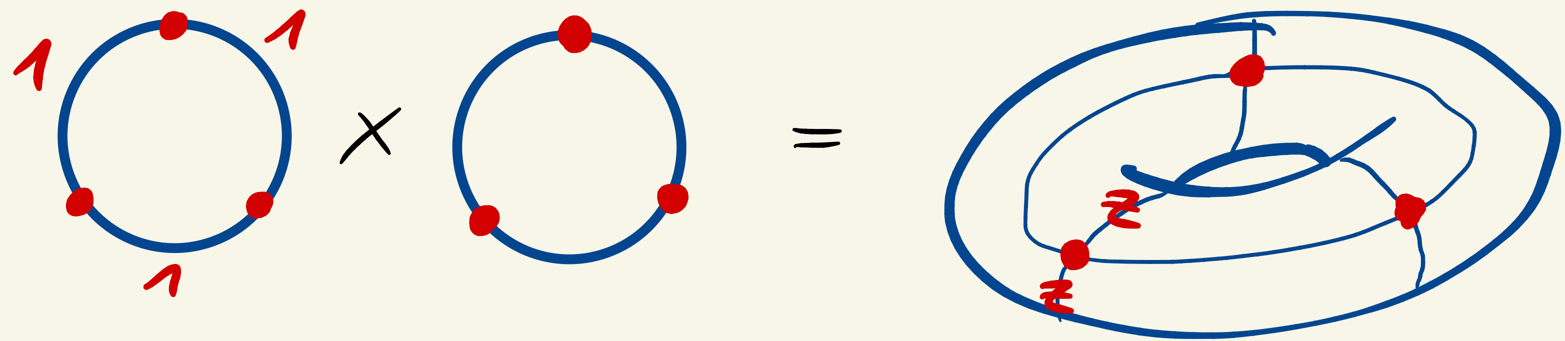
GOOD qLDPC CODES



product of classical rep. codes

toric code

GOOD qLDPC CODES



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INTUITION:

- size of quantum code given by input code lengths $N = n_1 \cdot n_2$

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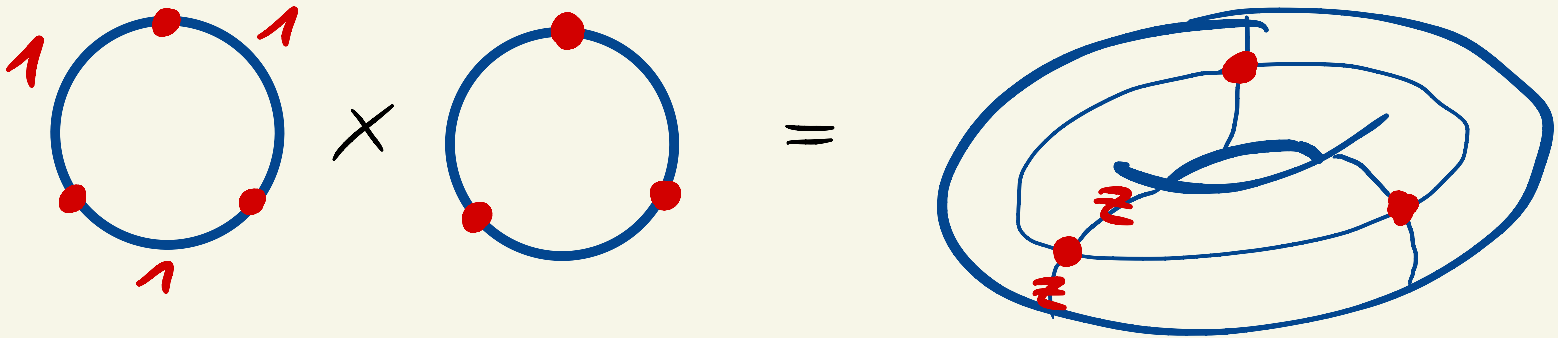

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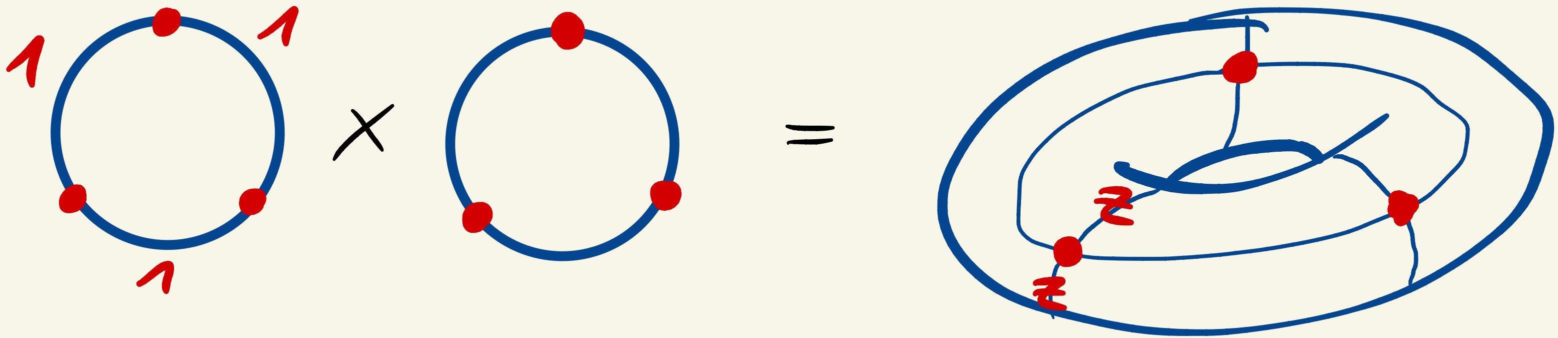

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toric code

INTUITION:

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- distance of product is $D = \min(d_1, d_2) = \Theta(\sqrt{N})$

GOOD qLDPC CODES

SLIDE FOR EXPERTS

ALGEBRAIC DESCRIPTION

Classical code $C = \{ \mathbb{F}_2^n \xrightarrow{H} \mathbb{F}_2^{n-k} \}$

GOOD qLDPC CODES

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GOOD qLDPC CODES

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edges

vertices

$$\begin{tikzcd} & \mathbb{F}_2^{n'} \arrow[r, "H'"] & \mathbb{F}_2^{n'-k'} \\ \mathbb{F}_2^{n} \arrow[d, "H"'] & & \\ \mathbb{F}_2^{n-k} & & \end{tikzcd}$$

Tensor product

$C \otimes C'$

GOOD qLDPC CODES

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faces

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vertical edges

vertices

horizontal edges

quantum code is 1st homology group

PARITY CHECKS: incidence between 1) edges & faces (H_Z)

2) edges & vertices (H_X)

GOOD qLDPC CODES

SLIDE FOR EXPERTS

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PARITY CHECKS: incidence between
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HOW CAN WE INCREASE THE DISTANCE?

GOOD qLDPC CODES

SLIDE FOR EXPERTS

ALGEBRAIC DESCRIPTION

Classical code $C = \{\mathbb{F}_2^{n} \xrightarrow{H} \mathbb{F}_2^{n-k}\}$

edges   vertices

balanced ~~Tensor~~ product

$C \otimes_G C'$

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balanced ~~Tensor~~ product

$C \otimes_G C'$

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Main Observation: if C & C' share a common symmetry G

$\Rightarrow$ can quotient out G

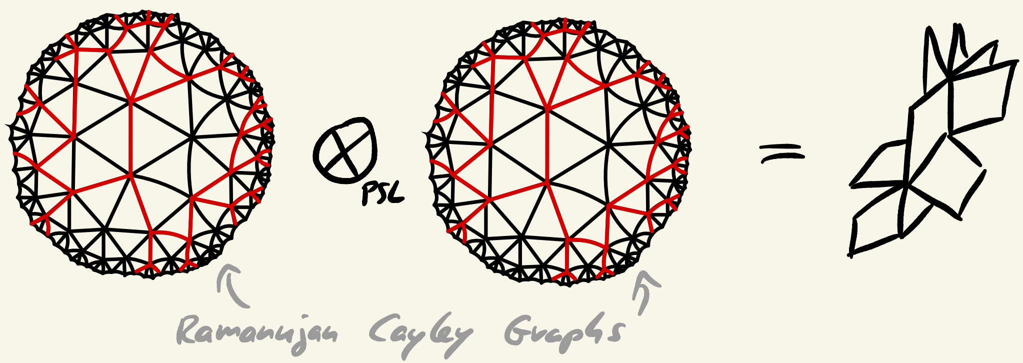
if $|G| = \Theta(n)$ $\rightarrow$ number of qubits stays linear after product

however homology stays unaffected $\Rightarrow d = \Theta(n)$

GOOD qLDPC CODES

CONJECTURE: NPB, EBERHARDT IEEE '21 & PRX '21

> Balanced products of expander codes $C(X,L)$ with $G = PSL_2(\mathbb{F}_q)$ & X Ramanujan graph give good qLDPC codes



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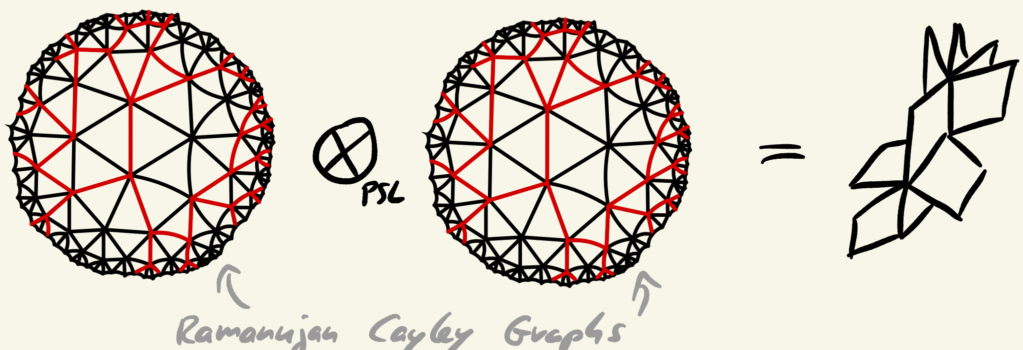

Proved by Panteleev & Kalachev for $C(X,L) \otimes_G C(X,K)^*$, SIGACT 54 '22

Similarly for $C(X,L) \otimes_G C(X,K) = C(X \times_G X, L \boxtimes K)$ [Dinur et al. '22]

Take $X \times_G X$ & do "rotated toric code version" [Leverrier-Zémor '22]

NLTS: PROOF SKETCH

Lower bound on circuit depth follows from the following Lemma:

Let $S_1, S_2 \subset \{0,1\}^n$ & D_ψ a prob. distr on $\{0,1\}^n$ obtained by measuring state $|\psi\rangle = U|0\ldots0\rangle$.

If $D_\psi(S_1), D_\psi(S_2) \geq \mu$, then the depth t of U is

$$t \geq \Omega\left(\log\left(\frac{\mathrm{dist}(S_1,S_2)^2\,\mu}{n}\right)\right)$$

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TASK: To prove NLTS we need to show that all low-energy states give rise to well-spread distributions.

NLTS: PROOF SKETCH

"Property 1" Clustering of approximate code words in CSS codes

fix constant $\delta > 0$, let $|y|_{C_X^\perp} = \min_{x \in C_X^\perp} |y + x|$

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$$\boxed{\begin{array}{l}\text{for } y \in \{0,1\}^n \text{ s.t. } |H_Z y| \leq \delta m \text{ we have either} \\ \qquad |y|_{C_X^\perp} \leq c_1 \varepsilon n \quad \text{or} \quad |y|_{C_X^\perp} \geq c_2 n\end{array}}$$

NLTS: PROOF SKETCH

"Property 1" Clustering of approximate code words in CSS codes

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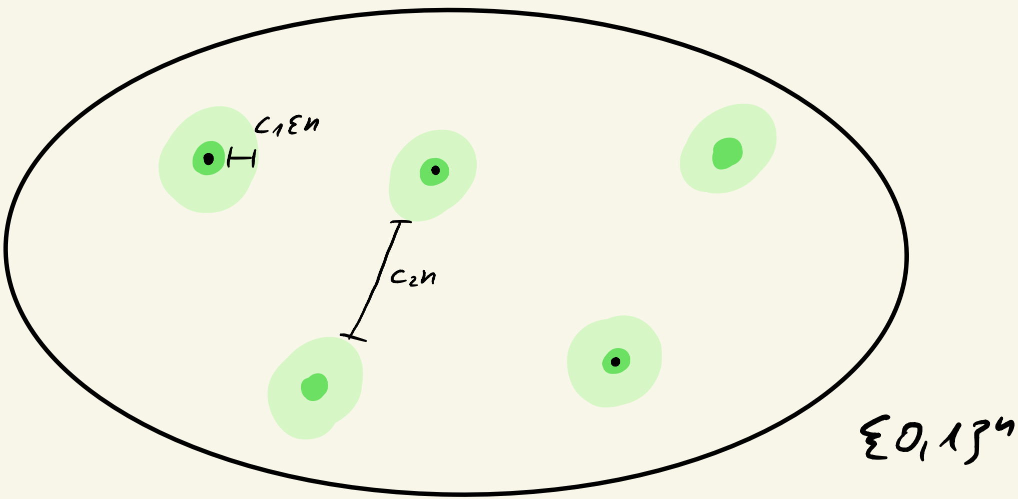

NLTS: PROOF SKETCH

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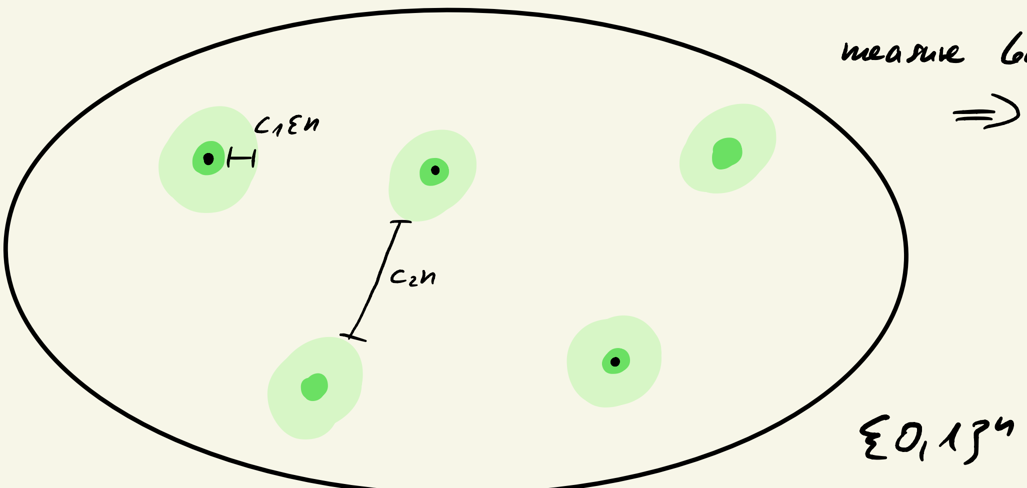

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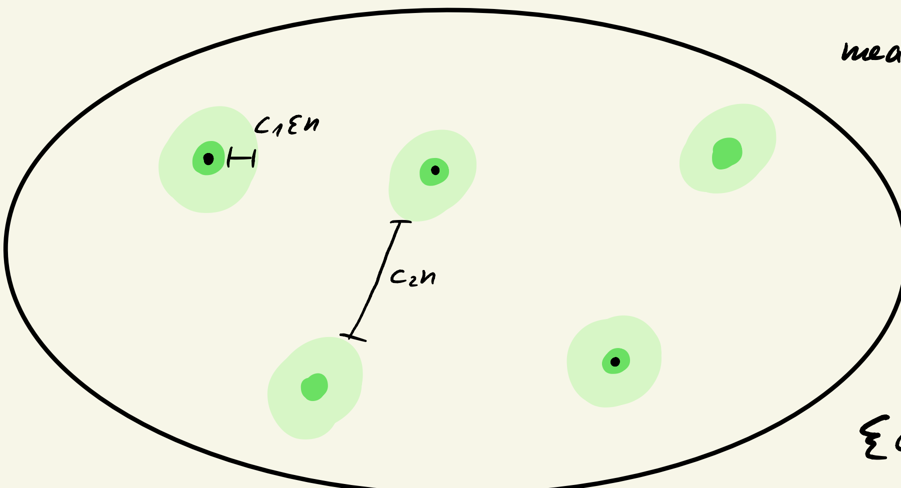

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NLTS: PROOF SKETCH

Left to show: distribution not concentrated on 1 cluster

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require $\varepsilon < O\left(\frac{k^2}{n^2}\right)$ $\Rightarrow$ need constant rate

$\boxtimes$

BEYOND NLTS

Quantum PCP conjecture

IT IS QMA-HARD TO DECIDE WHETHER THE GROUND STATE ENERGY OF A LOCAL HAMILTONIAN IS LESS THAN GIVEN NUMBER $a>0$ OR LARGER THAN $a+\Delta$ for $\Delta>0$ CONSTANT

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Statement about a computation (dynamic)

NLTS is about states (static)

BEYOND NLTS

Extend cNLTS: can use tensor network to encode quantum computation into Hamiltonian ground states

ANSHU, NPB, NGUYEN (to appear soon)

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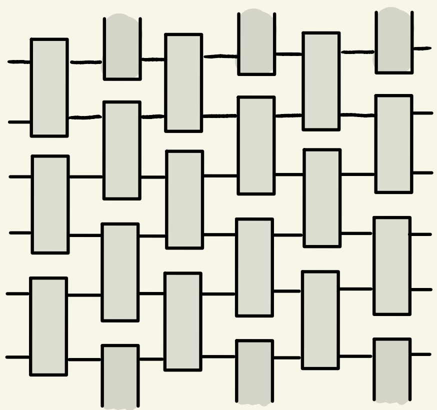

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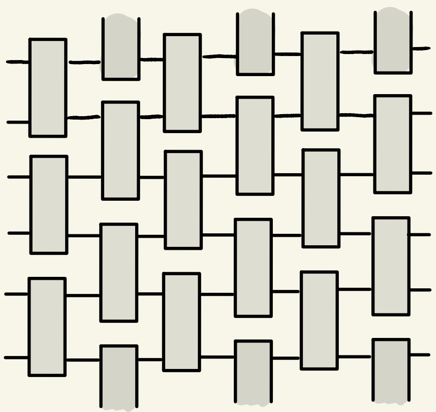
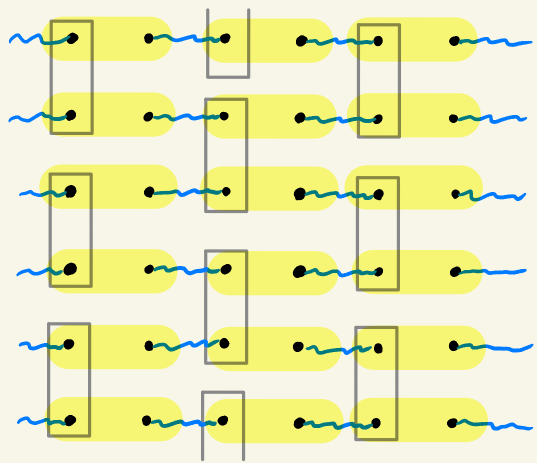


$\Longrightarrow$



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 $\hat{=}$ projector affecting computation (similar to MBQC)

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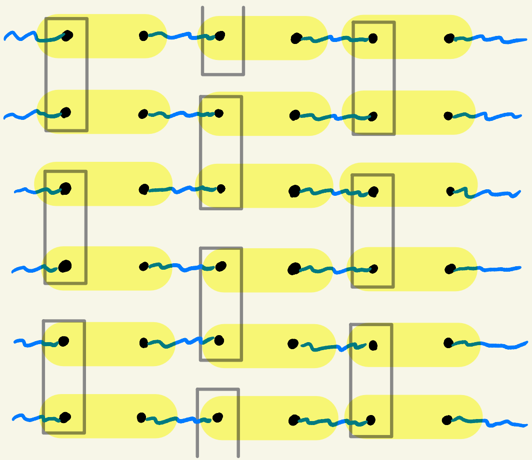

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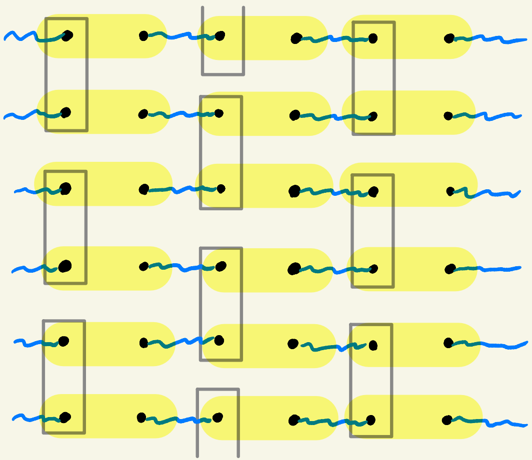

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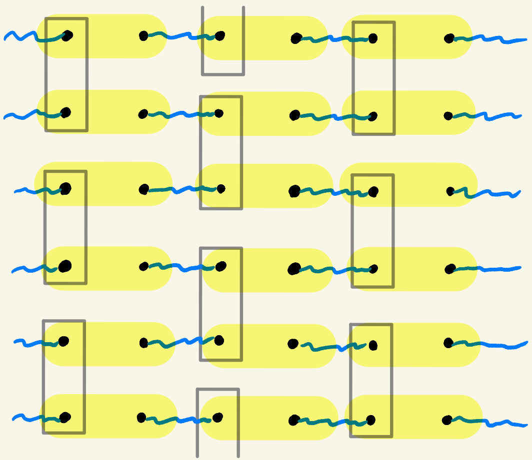

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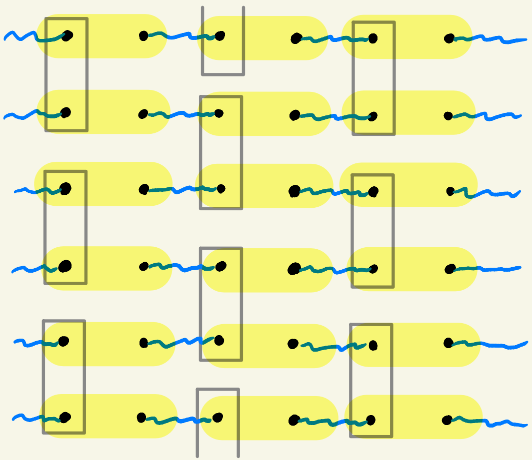

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↳ USE QUANTUM FAULT-TOLERANCE

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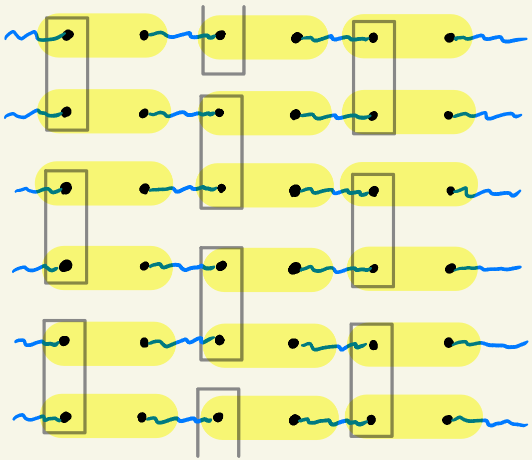
BEYOND NLTS

STEP TOWARDS PROVING POLYLOG-WEAKENED qPCP



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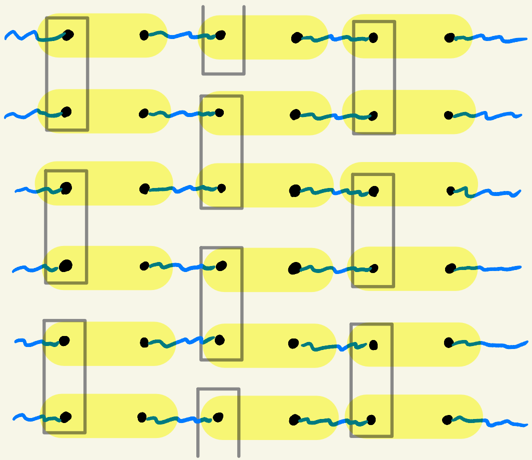

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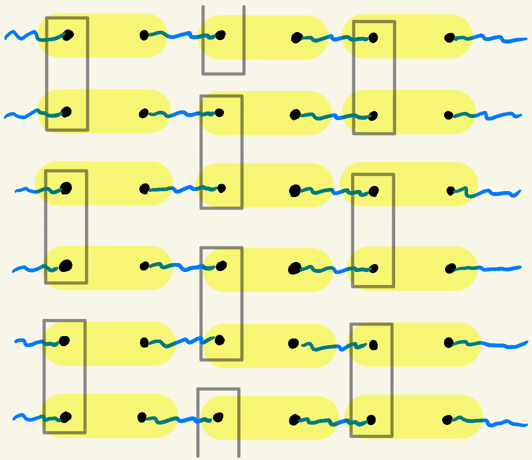

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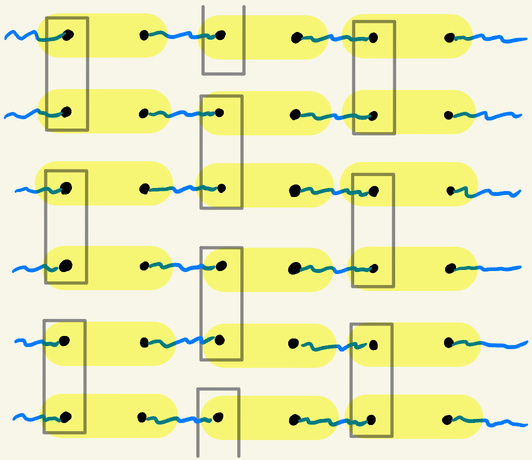

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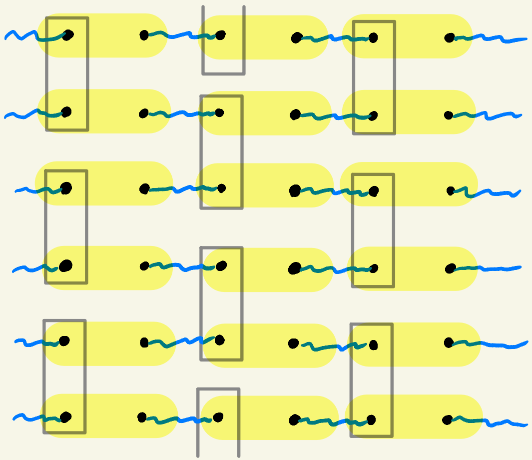

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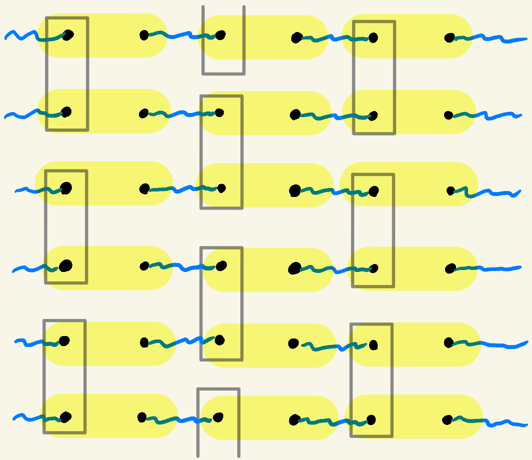

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SHOW THAT IF (*) ALSO HOLDS FOR $\frac{1}{\text{poly}(D)}$ - ENERGY STATES

& WE HAD FT-THEOREM AGAINST ADVERSARIAL NOISE

$\Rightarrow$ polylog-WEAK qPCP FOLLOWS

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THANK YOU!